MATH 554- 703 I - ANALYSIS I
SOLUTIONS FOR PROBLEMS 1-2 OF HOMEWORK ASSIGNMENT # 1

1. Using the field axioms, prove that for each $a \in F, a \neq 0$, the multiplicative inverse of a is unique.

   Solution: Let a be an arbitrary member of F and suppose that a has multiplicative inverses b and c, that is

   $$a\,b = 1 \tag{1}$$

   $$a\,c = 1 \tag{2}$$

   In this case, we can multiply both sides of equation (1) by c to obtain

   $$(a\,b)\,c = 1\,c$$

   Using the field axioms, this reduces to

   $$b\,(a\,c) = c$$

   and so $b = c$ using equation (2) and the identity axiom that $b\,1 = b$. $\square$

2. Using any of the results we proved in class (before the statement of this result) carefully prove that (-1)(-1)=1.

   Solution: We know from our previous results which were established in class that the additive inverse of the additive inverse of an element is the element itself

   $$-(-a) = a. \tag{3}$$

   We also know for each $a \in F$ that

   $$(-1)\,b = (-b)$$

   the additive inverse of b. Therefore applying this with $b = -1$ and using equation (3) we obtain

   $$(-1)(-1) = -(-1) = 1 \quad \square$$