MATH 554 - ANALYSIS I
FINAL EXAM – DEC. 12, 2001

1	(15 *pts*)
2	(15 *pts*)
3	(15 *pts*)
4	(15 *pts*)
5	(15 *pts*)
6	(15 *pts*)
7	(15 *pts*)
8	(15 *pts*)
9	(15 *pts*)
10	(15 *pts*)

Name: ____________________________
Code: ___________

Directions: To receive credit, you must justify your statements unless otherwise stated.

1. a.) Define the term "upper bound" of a set A.

   b.) Define the term "least upper bound" of a set A.

   c.) State the completeness **axiom** for the real numbers.

   d.) Prove: If γ is the least upper bound of a nonempty set A, then for each $\epsilon > 0$, there exists $a \in A$ such that
   $$\gamma - \epsilon < a \leq \gamma.$$

2. State and **sketch the proof** of the Archimedean Principle.

3. a.) Define what it means for a sequence $\{x_n\}_{n=1}^{\infty}$ to converge.

b.) Prove that $\left\{\frac{1}{n}\right\}_{n=1}^{\infty}$ converges to 0.

c.) Using the *properties* of limits, show that the sequence $\left\{\frac{n^3-1}{(n+1)^3}\right\}_{n=1}^{\infty}$ converges.

4. a). Define what it means for a set O to be "open".

b.) Give the definition of "open cover" $\mathcal{O}$ for a set K.

c.) Give the definition for a set K to be compact.

d.) State the Bolzanno-Weierstrass property for compact sets.

5. a.) State the $\epsilon - \delta$ definition of continuity of a function f at point x_0.

b.) State (at least) three equivalent conditions for a function $f : A \to \mathbb{R}$ to be continuous.

c.) Prove that the composition of two continuous functions is continuous. (You may use any properties of continuous functions that we have studied, but be certain to explain what you are doing.)

6. a.) Define "limit point" of a set.

b.) Prove that a compact set is closed.

7. Suppose that f is continuous on a compact set K, then prove that f is uniformly continuous.

8. State and **outline** the proof of the Heine-Borel Theorem.

9. a.) Define differentiability of a function f at a point x_0.

   b.) State and outline the proof of Rolle's Theorem.

10. a.) State both Parts I and II of the Fundamental Theorem of Calculus.

b.) Pick **one** of the two parts and sketch its proof.