MATH 527/CSCE 561 - NUMERICAL ANALYSIS
THURSDAY MARCH 30, 2006
TEST # 2

Directions: Answer all parts of each problem. As usual, show your work for full credit.

1. a) Determine the unique quadratic q so that $q(1) = 1$ and which vanishes at the two knots $\{-2, 3\}$.

   b) Consider the table

x	1	-2	3
y	3	1	-2

   Write the interpolating polynomial of minimal degree which fits the given data in its Lagrange form.

2. a) For the data

x	-2	0	1	2
y	-7	1	5	1

   use Newton's method to determine the interpolating polynomial $y = p(x)$ of minimal degree.

   b) Write the resulting Newton representation of the polynomial in its nested form.

   c) From the divided difference table, what is the value of $f[0, 1, 2]$?

3. Consider the polynomial given in Newton form:

$$p(x) = a_0 + \sum_{j=1}^{n} a_j \, \Pi_{k=0}^{j-1}(x - x_k)$$

   a) Write the psuedo code to efficiently evaluate the polynomial p, i.e. evaluate p in nested form.

   b) What is the computational complexity of this algorithm?

4. a) Use Taylor's formula to show

$$\frac{1}{2h}[4f(a+h) - 3f(a) - f(a+2h)] \;=\; f'(a) + \mathcal{O}(h^2)$$

   b) If $f(x) = 2e^{-2x}$ and $a = 0$, what value of h should be chosen above to guarantee an error from this procedure which does not exceed 10^{-4}.

5. Consider the integral $\int_{-1}^{3} (1 - x^2)\, dx$

   a) Compute the trapezoidal approximation for the partition $\{-1, 1, 3\}$.

   b) Perform one additional step of the recursive Trapezoidal approximation.

   c) Compute the Romberg approximation using these two estimates of the integral.

   d) Briefly indicate the idea for the error estimate for Romberg integration.

   For **extra credit of 15 points**, discuss briefly the computational complexity of the recursive Trapezoidal rule and Romberg integration.