Test # 1 Solns

#1 $(011\,010\,111)_2 = (327)_8 = (3\cdot 64 + 2\cdot 8 \cdot 7)_{10} = (215)_{10}$

$(.110\,100)_2 = (.64)_8 = \left(\frac{6}{8}+\frac{4}{64}\right)_{10} = (13/16)_{10}$

$\boxed{(327.64)_8}$ $\boxed{215\ 13/16 \text{ in base } 10}$

#2 standard form of quadratic formula is $x = \dfrac{-100 \pm \sqrt{9996}}{2}$.

(a) No loss of precision for $x_1 = \dfrac{-100 - \sqrt{9996}}{2}$

For $x_2 = \dfrac{-100 + \sqrt{9996}}{2}$ in finite precision arithemetic there will be a loss of precision due to the subtraction of two like quantities

$$\begin{array}{r} -100.00000 \\ 99.97998 \\ \hline 00.02002 \end{array}$$

loss of 3 leading significant digits

This is really Thm 1 of page 76 of text. There it is stated in terms of bits.

(b) Rewrite algebraically: $x_2 = \dfrac{-100+\sqrt{9996}}{2} \cdot \dfrac{100+\sqrt{9996}}{100+\sqrt{9996}}$

$$= \frac{-4}{2(100+\sqrt{9996})} = \boxed{-.0100010002}$$

#3 Suppose f has $n+1$ continuous derivatives on an interval $[a,x]$ then $f(x) = P_n(x) + E_n(x)$ where

$$P_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(x-a)^k$$

and

(a) $E_n(x) = \dfrac{f^{(n+1)}(\xi)}{(n+1)!}(x-a)^{n+1}$ for some ξ between a & x.

(b) $E_n(x) = \displaystyle\int_a^x f^{(n+1)}(t)\,\frac{(x-t)^n}{n!}\,dt$.

#4 $f^{(n)}(x) = \left(\frac{1}{2}\right)^n e^{1+x/2} \Rightarrow f^{(n)}(0) = \left(\frac{1}{2}\right)^n \cdot e$

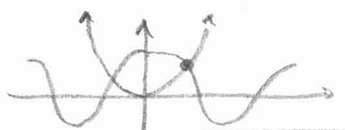
(a) $\boxed{P_4(x) = e + \frac{e}{2}x + \frac{e}{8}x^2 + \frac{e}{48}x^3}$

(b) $\boxed{E_n(x) = \frac{\left(\frac{1}{2}\right)^4 e^{1+\xi/2}}{4!} x^4 \text{, for some } \xi \text{ between } 0 \text{ \& } x.}$

(c) For all $|x|<1$, $|e^{1+\xi/2}| \le e^{3/2}$ so

$|E_n(x)| < \frac{e^{3/2}}{16 \cdot 24} x^4$. If $\frac{e^{3/2}}{384} x^4 \le 10^{-7}$, then the error estimate will be satisfied. This is the case if $\boxed{|x| \le .054}$

#5



a) $\boxed{\text{one positive soln}}$

b) If f is continuous on $[a,b]$ & $f(a) \cdot f(b) < 0$, then n applications of the bisection method guarantees that

$$|e_n| \le \frac{b-a}{2^{n+1}} \quad \text{where } e_n = x_n - r$$

& $f(r) = 0$

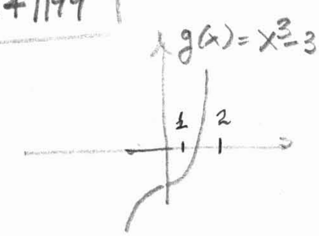
c) $f(x) = x^2 - \cos x$ $\quad f(0) = -1 < 0$, $f(1) = 1 - \cos(1) > 0$

$\boxed{a=0,\ b=1}$ $\quad x_0 = 1/2$, $x_1 = \frac{3}{4}$, $x_2 = 7/8$, $x_3 = \boxed{13/16}$

d) $|e_3| \le \frac{1}{2^4} = \boxed{\frac{1}{16}}$

#6 $f(x) = x^3$ Solve $f(x) = 3$ is the same as finding a zero for $g(x) = x^3 - 3$. Newton step: $\boxed{x_{n+1} = x_n - \frac{x_n^3 - 3}{3x_n^2}}$

a) $x_0 = 3$, $x_1 = 2.\overline{11}$, $x_2 = 1.631784$, $x_4 = 1.4341199$



b) $\max\limits_{[1,3]} |g''| = 6 \cdot 3 = 18$ $\quad C = \frac{1}{2} \frac{\max |f''|}{\min |f'|} = 3$

$\min\limits_{[1,3]} |g'| = 3$ $\quad \boxed{|e_{n+1}| \le 3 \cdot e_n^2}$

Grad Student Extra Problem – See proof in notes of Taylor's Thm