MATH 527/CSCE 561 - NUMERICAL ANALYSIS
TUESDAY FEBRUARY 14, 2006
TEST # 1

Directions: Work 5 out of the six. Mark the problem to be used for extra credit.

1. Compute the following base conversions.
   a) $(11010111.1101)_2 = ()_8 = ()_{10}$
   b) $(85.2)_{10} = ()_2$ [Note: Carry out to at most 9 places]

2. Consider solutions to the equation
$$x^2 + 100x + 1 = 0$$
in finite precision arithmetic.
   a) If you use the quadratic formula, how may digits of precision will you lose? Briefly explain or show why this happens.
   b) If there is a case with a loss of precision, how can the root be computed to full machine precision?

3. State Taylor's theorem expressing the error term
   a) in the standard form.
   b) as an integral.

4. Consider the function $f(x) = \exp(\frac{1}{2}x + 1)$.
   a) Compute the first four terms of the Taylor polynomial approximation of f about $c = 0$.
   b) Compute the error term.
   c) For which values of x is this Taylor approximant good to within an error of at most 10^{-7}?

5. Consider all positive solutions to the equation
$$x^2 - \cos(x) = 0 \tag{1}$$
   a) How many positive solutions are there?
   b) State the general theorem for the error estimate for the bisection method.
   c) Perform three iterations of the bisection method to approximate the smallest positive solution of this equation.
   d) What is the error estimate guaranteed by the theory for x_3.

6. a) Use Newton's method to approximate the cube root of 3 by solving $f(r) = 3$ with $f(x) := x^3$, using a starting value of $x_0 = 3$, and taking 3 iterations.
   b) Provide the error estimate guaranteed by the theory.