Math 527/CSCE 561 - Numerical Analysis
Thursday March 30, 2006
Sample Problems for Test # 2

Directions: This test is purposefully a little longer in order to give you the best idea of the topics covered. I could also ask you to write down some of the pseudo code we have done for the algorithms.

1. Consider the following data:

x	0	2	3
y	-1	9	17

a) Using Newton's method, determine the polynomial $y = p(x)$ of minimal degree which interpolates this data.

b) Write the resulting polynomial in nested form.

c) In general, what is the computational complexity of evaluation of a polynomial given in nested form?

d) Write the interpolating polynomial in Lagrange form.

2. Consider the function $f(x) = 8x^4 + 3x^3 + 2x + 3$.

a) Define the divided difference $f[x_0, x_1, \ldots, x_n]$.

b) Use a divided difference table to interpolate f at the points $\{-\frac{1}{2}, 0, \frac{1}{2}, 1\}$.

3. a) Derive the finite difference formula:

$$\frac{f(a+h) - 2f(a) + f(a-h)}{h^2} = f''(a) + f^{(4)}(a)\frac{h^2}{12} + f^{(6)}(\xi)\frac{h^4}{360}$$

b) Can Richardson extrapolation be used to obtain a 4-th order scheme to approximate the second derivative of f?

4. Consider the integral $\int_0^4 2^x \, dx$

a) Compute the trapezoidal approximation for the partition $\{0, 2, 4\}$.

b) Perform one additional step of the recursive Trapezoidal approximation.

c) Compute the Romberg approximation with these partitions.

5. a) Verify the centered difference formula

$$\phi(a+H) - \phi(a-H) = 2H\phi'(a) + \frac{1}{3}\phi''(\xi)H^3$$

b) Set by $\phi(x) = \int_0^x f(t)\, dt$ and use part a) with $a = h/2$ and $H = h/2$ to obtain the *midpoint* quadrature rule

$$\int_0^h f(t)\, dt = h\, f(h/2) \; + \; \frac{h^3}{24} f''(\xi).$$

Extra Credit: Using part b) in the last problem, establish the *composite midpoint rule*:

$$\int_a^b f(t)\,dt \;=\; h\sum_{j=1}^{n} f(a+\frac{2j-1}{2}h)) \;+\; \frac{(b-a)\,f''(\xi)}{24}\,h^2.$$