Math 527/CSCE 561
(to begin Tuesday Lecture)

We are attempting to see how accuracy and efficiency are intertwined in the case of Taylor polynomial approximations. The example is to compute $\ln 2$ correct to 8 significant digits using Taylor polynomials applied to $f(x) := \ln(1+x)$ based around $a = 0$.

As computed in class, the Taylor approximant of degree n is

$$P_n(x) = \sum_{j=1}^{n} (-1)^{j-1} \, \frac{x^j}{j}$$

The second form of the error term is

$$E_n(x) = (-1)^n \, \frac{1}{(1+\xi)^n} \, \frac{x^{n+1}}{n+1}$$

where ξ is an unknown quantity some place between 0 and 1, and so $0 < 1+\xi < 2$. Therefore, to make this error term small

$$\frac{1}{(1+\xi)^n} \, \frac{x^{n+1}}{n+1} < .5 \times 10^{-8}$$

with $x = 1$ and not knowing the value of ξ, we should choose n large enough to guarantee

$$\frac{1}{n+1} < .5 \times 10^{-8}$$

and therefore insure that $\ln 2$ is approximated to 8 significant digits.

What is the degree of the polynomial to do this?

How many computations must be made, at a minimum, to perform the approximation in this way?