Homework solutions #7

4.1.15. Natural thing to try would be to create an interpolating polynomial using Newton's method. 4th (and hence, 5th) column in the divided difference table would be zero. The polynomial would then be

$$p_5(x) = 1 + 3(x+2) + 2(x+2)(x+1) + \underline{(x+2)(x+1)(x)}$$

of third degree. This suggests that data came from the cubic polynomial - though, not necessary - the underlying function may be not of the polynomial nature at all. If it is, then it is $p_5(x)$.

4.1.19. $p_4(x) = -1 + 2(x+2) - (x+2)(x+1) + (x+2)(x+1)x$

$p_2(x) = 1 + 2(x+1)x$

-2	-1				
		2			
-1	1		-1		
		0		+1	
0	1		2		0
		4		1	
1	5		5		
		14			
2	19				

for P_4

Note that in triangle there's a div. diff. table for p_2.

4.1.23

1	3			
		2		
2	5		-1	
		0		∞
3	5		1	
		-1		
1	7			

Newton's computations would require division by zero, caused by having two different values for $f(1)$. Function cannot have 2 values for one argument - "vertical line test".

4.1.26 Using inverse interpolation, switch data sets:

-31	-2				
		0.27			
5	-1		-0.016		
		-0.5		0.027	
1	1		.1		-0.0000498
		0.1		-0.018	
11	2		-0.013		
		0.02			
61	3				

This yields certain polynomial $f^{-1}(y)$. Plugging $y=0$ gives $x = 1.932$.

Now it's important to look into <u>what happened</u>. Expected zero was supposed to lie between -2 and -1 - because function $f(x)$ changes the sign there. It is still possible there's a zero on $[1,2]$ but very much less likely - and <u>not expected</u>. We may conclude the inverse interpolation method did a poor job here.

4.1.27 $p(x) = ax^3 + bx^2 + cx + d$, where a, b, c, d - to be defined.

$$\left.\begin{aligned} p(0) &= d = 1 \\ p(1) &= a+b+c+d = 0 \\ p'(0) &= 3ax^2 + 2bx + c\big|_{x=0} = c = 0 \\ p'(-1) &= 3a - 2b + c = -1 \end{aligned}\right\}$$

this gives a system of equations

$$\begin{cases} a+b = -1 \\ 3a-2b = -1 \\ c = 0 \\ d = 1 \end{cases}$$

Solving the system, $a = -\frac{3}{5}$, $b = -\frac{2}{5}$

$$p(x) = -\frac{3}{5}x^3 - \frac{2}{5}x^2 + 1.$$

4.1.34, 35 - Pure algebra, MATH 111.

4.1.36 : $p_2(x) = \underbrace{f_0}_{} + \underbrace{f[x_0,x_1]}_{} (x-x_0) + \underbrace{f[x_0,x_1,x_2]}_{} (x-x_0)(x-x_1)$.

these are just numbers

$$p_2'(x) = f[x_0,x_1] + f[x_0,x_1,x_2]\left(2x - (x_0+x_1)\right)$$

$$p_2''(x) = 2f[x_0,x_1,x_2].$$

4.1.37.

$$f[x_0,x_1] = \frac{f_1 - f_0}{x_1 - x_0} = \frac{\Delta f_0}{h}$$

$$f[x_0,x_1,x_2] = \frac{f[x_1,x_2] - f[x_0,x_1]}{x_2 - x_0} = \left\{ \begin{array}{l} \text{substitute } \frac{\Delta f}{h} \text{ into num.,} \\ x_2 - x_0 = 2h \text{ into denom.} \end{array} \right\}$$

$$= \frac{\frac{\Delta f_1}{h} - \frac{\Delta f_0}{h}}{2h} = \frac{\Delta f_1 - \Delta f_0}{2h^2} = \frac{\Delta^2 f_0}{2h^2}.$$

4.1.47.

Using Maple could have helped checking the equality of 2 expressions.

a) $f[x_0 x_1] = \frac{f_1 - f_0}{x_1 - x_0}$, $\begin{vmatrix} 1 & f_0 \\ 1 & f_1 \end{vmatrix} = f_1 \cdot 1 - f_0 \cdot 1 = f_1 - f_0$

$\begin{vmatrix} 1 & x_0 \\ 1 & x_1 \end{vmatrix} = \quad = x_1 - x_0$

qed