Homework #2: Solutions

~~2.3.1~~

2.22b $0.125 = 2^{-3} \Rightarrow b_1 = 0$

$$(b_2 \dots b_9)_2 - 127 = -3$$

$$(b_{10} \dots b_{32})_2 = 0.$$

$$(b_2 \dots b_9)_2 = 124 \Rightarrow (b_2 \dots b_9)_2 = (01111100)_2.$$

$$(0.125)_{10} = (0\,\underbrace{0\,1\,1\,1\,1\,1\,0\,0}\,\underbrace{0\,0 \dots 0})_{MARC-32}.$$

$$-0.125 = -1 \cdot 2^{-3} \Rightarrow b_1 = 1.$$

$$(-0.125)_{10} = (1\,\underbrace{0\,1\,1\,1\,1\,1\,0\,0}\,\underbrace{0\,0 \dots 0})_{MARC-32}$$

2.3.3 $e^x - e = e(e^{x-1} - 1) = e\left[\left(1 + (x-1) + \frac{(x-1)^2}{2} + \right.\right.$

$\left.\left. + \frac{(x-1)^3}{3!} + \text{etc till enough precision}\right) - 1\right] =$

$= e\left[x - 1 + \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3!} + \dots\right].$

When to use Taylor approximation? By theorem on loss of precision, restrict loss of significant digits by $p = 1$.

$$\frac{1}{2} \leq 1 - \frac{e^x}{e} \iff$$

$$e^{x-1} \leq \frac{1}{2} \iff$$

$$x - 1 \leq \ln\frac{1}{2}.$$

2.3.7 $y = \frac{1}{x}(\sinh x - \tanh x)$. ~~Mult~~

$y = \frac{1}{x} \cdot \sinh x \cdot \left(1 - \frac{1}{\cosh x}\right)$. Multiply by "adjoint" term:

$$y = \frac{1}{x} \cdot \sinh x \left(1 - \frac{1}{\cosh x}\right) \cdot \frac{\left(1 + \frac{1}{\cosh x}\right)}{\left(1 + \frac{1}{\cosh x}\right)} =$$

$$= \frac{1}{x} \sinh x \cdot \frac{1 - \frac{1}{\cosh^2 x}}{1 + \frac{1}{\cosh x}} = \frac{\sinh x}{x\left(1 + \frac{1}{\cosh x}\right)} \cdot \frac{\cosh^2 x - 1}{\cosh^2 x} =$$

$$= \frac{\sinh^3 x}{x(\cosh^2 x + \cosh x)}.$$

2.3.11 $f(x) = \ln\left(x + \sqrt{x^2+1}\right)$. When x is negative,

$\sqrt{x^2+1} \sim -x$, therefore cancellation may occur.

$$f(x) = \ln\left(\left(x+\sqrt{x^2+1}\right)\cdot\frac{-x+\sqrt{x^2+1}}{-x+\sqrt{x^2+1}}\right) = \ln\left(\frac{x^2+1-x^2}{\sqrt{x^2+1}-x}\right) =$$

$$= \ln\frac{1}{\sqrt{x^2+1}-x} = \ln\left[\left(\sqrt{x^2+1}-x\right)^{-1}\right] = -\ln\left(\sqrt{x^2+1}-x\right).$$

These identities are good for all x but we're going to use them, when $x<0$, since $\sqrt{x^2+1}-x$ will actually be addition of positive numbers.

5h. $(01111011)_2 = 123; \quad 123-127=-4$

$(1.10011001100110011001100)_2 \cdot 2^{-4} =$

$= (0.\underbrace{000}_{0}\underbrace{110}_{6}\underbrace{011}_{3}\underbrace{001}_{1}\underbrace{100}_{4}\underbrace{110}_{6}\underbrace{011}_{3}\underbrace{001}_{1}\underbrace{100}_{4})_2 =$

$= (0.063146314)_8 =$

$= 8^{-9} \cdot (63146314)_8 = 13421772/134217728 =$

$\simeq 0.09999999403954_{10}.$

5e. $(0\ 00000001\ 0..0)_2 = (-1)^0 \cdot 2^{1-127} \times (1,0..0) =$

$= \frac{1}{2^{126}}$

5b. $(1\ 00000000\ 0..0)$ = reserved for (-0).

2.3 1 When x is small, $\sqrt{x+4} \sim 2$, therefore the subtraction may cause loss of significance. To avoid it, ~~we~~ rationalize the numerator:

$$f(x) = \sqrt{x+4} - 2 = \left(\sqrt{x+4} - 2\right) \cdot \frac{\sqrt{x+4}+2}{\sqrt{x+4}+2} =$$

$$= \frac{(x+4)-4}{\sqrt{x+4}+2} = \frac{x}{\sqrt{x+4}+2}.$$ No more subtraction!