Homework #1 : Solutions

absolute error | relative error

1.1.1 $\left|\frac{22}{7}-\pi\right| = 0.001264489$ $\qquad$ $\frac{\left|\frac{22}{7}-\pi\right|}{\pi} = 0.000402499$

$\left|\frac{355}{113}-\pi\right| = 2.66\cdot 10^{-7}$ $\qquad$ $\left|\frac{\frac{355}{113}-\pi}{\pi}\right| = 8.467\cdot 10^{-8}$

for example of other fractions, the most trivial one would be $\frac{3141592654}{1000000000}$.

2. $\left|\frac{x-0.6032}{x}\right| = 0.001 \iff$

OR $\begin{cases} x-0.6032 = 0.001x \\ x-0.6032 = -0.001x \end{cases} \iff$

$\begin{cases} 0.999x = 0.6032 \\ 1.001x = 0.6032 \end{cases} \iff \begin{cases} x = \frac{6032}{999} \\ x = \frac{6032}{1001} \end{cases}$

3. $\left|\frac{5.000-4.9997}{4.9997}\right| \simeq 0.00006 = 6\cdot 10^{-5}$

10. $z = \sum_{i=1}^{n}\left(b_i^{-1}\prod_{j=1}^{i} a_j\right)$, e.g. $n=1$: $\frac{1}{b_1}\cdot a_1$

$n=2$ $\quad \frac{1}{b_1}a_1 + \frac{1}{b_2}a_1 a_2$.

The most common part is a_1:

10...

$$a_1\left(\frac{1}{b_1}+a_2\left(\frac{1}{b_2}+\ldots+a_{n-1}\left(\frac{1}{b_{n-1}}+\frac{a_n}{b_n}\right)\right)\right).$$

$z=\frac{a_n}{b_n}$

for $i=n-1$ downto 1
  $z=\left(z+\frac{1}{b_i}\right)a_i$
end for

OR

for $i=1$ to $n-1$
  $z=\left(z+\frac{1}{b_{n-i}}\right)\cdot a_{n-i}$
end for

The other option would be to carry product of a's with you in a temporary variable:

$z=0$
$t=1$
for $i=1$ to n
  $t=t\cdot a_i$
  $z=z+t/b_i$
end for

11d. $p(t) = \sum_{i=1}^{n} \left[a_i \prod_{j=1}^{i-1} (t - x_j) \right]$

$n = 1: \quad p(t) = a_1 \cdot 1$

$n = 2: \quad p(t) = a_1 \cdot 1 + a_2 \cdot (t - x_1)$

$n = 3: \quad p(t) = a_1 \cdot 1 + a_2 \cdot (t - x_1) + a_3 \cdot (t - x_1)(t - x_2)$.

The most common term is $(t - x_1)$:

$$p(t) = a_1 + (t - x_1)\Big(a_2 + (t - x_2)\big(a_3 + \ldots + (t - x_{n-1}) a_n\big)\Big)$$

$p = a_n$

for $i = n-1$ downto 1

  $p = a_i + (t - x_i)\, p$

end

OR with a temporary variable:

$$p(t) = a_1 \cdot \underbrace{1}_{} + a_2 \underbrace{(t - x_1)}_{} + a_3 \underbrace{(t - x_1)(t - x_2)}_{} + \ldots + a_n \underbrace{(t - x_1)(t - x_2) \cdots (t - x_{n-1})}_{}$$

$q \cdot (t - x_1) \qquad q \cdot (t - x_2) \qquad q \cdot (t - x_{n-1})$:

$p = 0$

$q = 1$

for $i = 1$ to n

  $= a_i \cdot q + p$

  $q = q \cdot (t - x_i)$

end

12a. $n=0$: $v = a_0$

$n=1$: $v = a_0 + X \cdot a_1$

$n=2$: $v = a_0 + X \cdot a_1 + X \cdot a_2$.

$$v = a_0 + \sum_{i=1}^{n} a_i \cdot X$$

14. The only multiplication occurs inside of nested loop: The total count would be
$1+2+3+\dots+n = \frac{n\cdot(n+1)}{2}$.

1.1c.20 a): $x = real(m/n)$: first we do integer division, then cast the result to be real.

$y = real(m)/real(n)$: here we divide numbers already casted up to reals, so the division doesn't truncate the remainder.

for example, $m=4$, $n=2$: $\frac{4}{2} = 2 = \frac{4.0}{2.0}$ but

$m=5$, $n=2$ $\frac{5}{2} = 2$, rem. 1

$\frac{5.0}{2.0} = 2.5$

b) Same thing here: $n_1 = 11$; $n_2 = 10$.