Math 522, Homework 7

Here is a hint on Problem 8 (b), but you will need to fill in the details:

Since, the Fourier transform

$$\widehat{\chi}_{[-\frac{1}{2},\frac{1}{2}]}(\lambda) = \frac{1}{\sqrt{2\pi}}\, sinc(\lambda/2)$$

where $sinc(\lambda) := \sin(\lambda)/\lambda$, then using the dilation property of the Fourier transform it follows that

$$Sinc(\lambda) = \frac{1}{\sqrt{2\pi}}\ \widehat{\chi}_{[-\pi,\pi]}(\lambda)$$

If it helps you to see how the reasoning goes more easily, this last inequality (along with the Fourier inversion theorem) shows

$$\widehat{Sinc}(\lambda) = \frac{1}{\sqrt{2\pi}}\, \chi_{[-\pi,\pi]}(\lambda)$$

To show $Sinc$ is a scaling function, you will need to show that

$$\langle\ Sinc(t-k), Sinc(t-\ell)\ \rangle = \delta_{k,\ell}$$

The hint is to establish the following identity by using Plancherel's formula, which was covered last lecture, the above identities for $Sinc$ and the translation property for the Fourier transform:

$$\int_{-\infty}^{\infty} Sinc(\lambda-k)\ Sinc(\lambda-\ell)\, d\lambda = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{-ikt}\ \overline{e^{-i\ell t}}\, dt$$