In Example 2.1-6, the computations are very easy because there is no difficulty in the determination of the appropriate values of h and m. However, instead of drawing only one card, suppose that 13 are taken at random and without replacement. We can think of each possible 13-card hand as being an outcome in a sample space, and it is reasonable to assume that each of these outcomes has the same probability. To use the above method to assign the probability of a hand, consisting of seven spades and six hearts, for illustration, we must be able to count the number h of all such hands as well as the number m of possible 13-card hands. In these more complicated situations, we need better methods of determining h and m. We discuss some of these counting techniques in Section 2.2.

EXERCISES 2.1

2.1-1 Draw one card at random from a standard deck of cards. The sample space S is the collection of the 52 cards. Assume that the probability set function assigns 1/52 to each of these 52 outcomes. Let

$$A = \{x : x \text{ is a jack, queen, or king}\},$$

$$B = \{x : x \text{ is a 9, 10, or jack and } x \text{ is red}\},$$

$$C = \{x : x \text{ is a club}\},$$

$$D = \{x : x \text{ is a diamond, a heart, or a spade}\}.$$

Find (a) $P(A)$, (b) $P(A \cap B)$, (c) $P(A \cup B)$, (d) $P(C \cup D)$, (e) $P(C \cap D)$.

2.1-2 A coin is tossed four times, and the sequence of heads and tails is observed.

(a) List each of the 16 sequences in the sample space S.

(b) Let events A, B, C, and D be given by $A = \{\text{at least 3 heads}\}$, $B = \{\text{at most 2 heads}\}$, $C = \{\text{heads on the third toss}\}$, and $D = \{\text{1 head and 3 tails}\}$. If the probability set function assigns 1/16 to each outcome in the sample space, find (i) $P(A)$, (ii) $P(A \cap B)$, (iii) $P(B)$, (iv) $P(A \cap C)$, (v) $P(D)$, (vi) $P(A \cup C)$, and (vii) $P(B \cap D)$.

2.1-3 A field of beans is planted with three seeds per hill. For each hill of beans, let A_i be the event that i seeds germinate, $i = 0, 1, 2, 3$. Suppose that $P(A_0) = 1/64$, $P(A_1) = 9/64$, and $P(A_2) = 27/64$. Give the value of $P(A_3)$.

2.1-4 Consider the trial on which a 3 is first observed in successive rolls of a four-sided die. Let A be the event that 3 is observed on the first trial. Let B be the event that at least two trials are required to observe a 3. Assuming that each side has probability 1/4, find (a) $P(A)$, (b) $P(B)$, and (c) $P(A \cup B)$.

2.1-5 A fair eight-sided die is rolled once. Let $A = \{2, 4, 6, 8\}$, $B = \{3, 6\}$, $C = \{2, 5, 7\}$, and $D = \{1, 3, 5, 7\}$. Assume that each face has the same probability.

(a) Give the values of (i) $P(A)$, (ii) $P(B)$, (iii) $P(C)$, and (iv) $P(D)$.

(b) Give the values of (i) $P(A \cap B)$, (ii) $P(B \cap C)$, and (iii) $P(C \cap D)$.

(c) Give the values of (i) $P(A \cup B)$, (ii) $P(B \cup C)$, and (iii) $P(C \cup D)$ using Theorem 2.1-5.

2.1-6 If $P(A) = 0.4$, $P(B) = 0.5$, and $P(A \cap B) = 0.3$, find (a) $P(A \cup B)$, (b) $P(A \cap B')$, and (c) $P(A' \cup B')$.

2.1-7 If $S = A \cup B$, $P(A) = 0.7$, and $P(B) = 0.9$, find $P(A \cap B)$.

2.1-8 If $P(A) = 0.4$, $P(B) = 0.5$, and $P(A \cup B) = 0.7$, find

(a) $P(A \cap B)$.

(b) $P(A' \cup B')$.

2.1-9 Roll a fair six-sided die three times. Let $A_1 = \{1 \text{ or } 2 \text{ on the first roll}\}$, $A_2 = \{3 \text{ or } 4 \text{ on the second roll}\}$, and $A_3 = \{5 \text{ or } 6 \text{ on the third roll}\}$. It is given that $P(A_i) = 1/3$, $i = 1, 2, 3$; $P(A_i \cap A_j) = (1/3)^2$, $i \neq j$; and $P(A_1 \cap A_2 \cap A_3) = (1/3)^3$.

(a) Use Theorem 2.1-6 to find $P(A_1 \cup A_2 \cup A_3)$.

(b) Show that $P(A_1 \cup A_2 \cup A_3) = 1 - (1 - 1/3)^3$.

2.1-10 Prove Theorem 2.1-6.

2.1-11 For each positive integer n, let $P(\{n\}) = (1/2)^n$. Consider the events $A = \{n : 1 \leq n \leq 10\}$, $B = \{n : 1 \leq n \leq 20\}$ and $C = \{n : 11 \leq n \leq 20\}$. Find (a) $P(A)$, (b) $P(B)$, (c) $P(A \cup B)$, (d) $P(A \cap B)$, (e) $P(C)$, and (f) $P(B')$.

2.1-12 Let x equal a number that is selected randomly from the closed interval from zero to one, that is $[0, 1]$. Use your intuition to assign values to

(a) $P(\{x : 0 \leq x \leq 1/3\})$.

(b) $P(\{x : 1/3 \leq x \leq 1\})$.

(c) $P(\{x : x = 1/3\})$.

(d) $P(\{x : 1/2 < x < 5\})$.

✓ **2.1-13** A typical roulette wheel used in a casino has 38 slots that are numbered $1, 2, 3, \ldots, 36$, 0, 00, respectively. The 0 and 00 slots are colored green. Half of the remaining slots are red and half are black. Also half of the integers between 1 and 36 inclusive are odd, half are even, and 0 and 00 are defined to be neither odd nor even. A ball is rolled around the wheel and ends up in one of the slots; we assume each slot has equal probability of 1/38 and we are interested in the number of the slot in which the ball falls.

(a) Define the sample space S.

(b) Let $A = \{0, 00\}$. Give the value of $P(A)$.

(c) Let $B = \{14, 15, 17, 18\}$. Give the value of $P(B)$.

(d) Let $D = \{odd\}$. Give the value of $P(D)$.

2.1-14 The five numbers 1, 2, 3, 4, and 5 are written, respectively, on five disks of the same size and placed in a hat. Two disks are drawn without replacement from the hat, and the numbers written on them are observed.

(a) List the 10 possible outcomes for this experiment as pairs of numbers (order not important).

(b) If each of the 10 outcomes has probability 1/10, assign a value to the probability that the sum of the two numbers drawn is (i) 3; (ii) between 6 and 8 inclusive.

2.1-15 Divide a line segment into two parts by selecting a point at random. Use your intuition to assign a probability to the event that the larger segment is at least two times longer than the shorter segment.

2.1-16 Let the interval $[-r, r]$ be the base of a semicircle. If a point is selected at random from this interval, assign a probability to the event that the length of the perpendicular segment from this point to the semicircle is less than $r/2$.