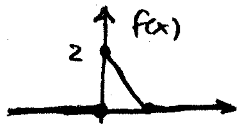
Math/Stat 511 HW #9 3/31/01

§4.1

#1 $f(x) = \begin{cases} 2(1-x), & 0 \le x \le 1 \\ 0, & \text{elsewhere} \end{cases}$

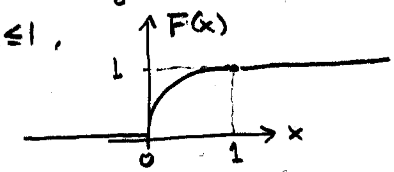
(a) Sketch of pdf



(b) Cumulative distribution

$F(x) = \begin{cases} 0, & x < 0 \\ x(2-x), & 0 \le x \le 1 \\ 1, & x \ge 1 \end{cases}$

since $F(x) = \int_0^x 2(1-t)\,dt = -(1-t)^2 \Big|_0^x$ if $0 \le x \le 1$,



(c) (i) $P(0 \le X \le 1/2) = F(\tfrac{1}{2}) - F(0) = \boxed{3/4}$

(ii) $P(\tfrac{1}{4} \le X \le \tfrac{3}{4}) = F(\tfrac{3}{4}) - F(\tfrac{1}{2}) = \boxed{\tfrac{1}{2}}$

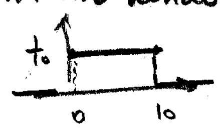
(iii) $P(X = \tfrac{3}{4}) = \int_{3/4}^{3/4} f(x)\,dx = \boxed{0}$, (iv) $P(X \ge 3/4) = 1 - P(X \le \tfrac{3}{4}) = \boxed{\tfrac{1}{16}}$

#8 $f(x) = \begin{cases} c/x^2, & x \ge 1 \\ 0, & \text{otherwise} \end{cases}$ (a) For f to be a pdf, $\int f\,dx = 1$

so $1 = c\int_1^\infty \frac{dx}{x^2} = c \cdot \lim_{N\to\infty} \left(-\frac{1}{x}\right)\Big|_1^N = c \cdot 1 \Rightarrow \boxed{c = 1}$

(b) $E[X] = \int_1^\infty x \frac{1}{x^2}\,dx = \lim_{N\to\infty} \ln x \Big|_1^N = \lim_{N\to\infty} (\ln N) = \boxed{+\infty}$

§4.2 **#3** The problem specificies that X is a uniform random variable on $[0,10]$. (a) $\boxed{f(x) = \begin{cases} \frac{1}{10}, & 0 \le x \le 10 \\ 0, & \text{otherwise} \end{cases}}$



(b) $P(X \ge 8) = \int_8^{\infty} f(x)\,dx = \frac{1}{10}\int_8^{10} dx = \boxed{\tfrac{1}{5}}$, (c) $P(2 \le X < 8) = \int_2^8 f\,dx = \boxed{\tfrac{3}{5}}$

(d) $E[X] = \int_0^{10} x \frac{1}{10}\,dx = \frac{1}{10} \frac{x^2}{2}\Big|_0^{10} = \boxed{5}$ (e) $\sigma^2 = \int_0^{10} x^2 \frac{1}{10}\,dx - (5)^2 = \boxed{\tfrac{25}{3}}$

#9(a) From our work in this section, we recognize $M(t) = \frac{1}{1-3t}$ as the moment generating function of the exponential distribution with parameter $\theta = 3$ ($M_X(t) = \frac{1}{1-\theta t}$, $t < \frac{1}{\theta}$ if X is distibuted as exponential with parameter θ). Since this transform is 1:1, then

$$\boxed{f(x) = \frac{1}{3} e^{-x/3}, \quad 0 < x < \infty}$$

In this case $\boxed{\mu = \theta = 3}$ & $\boxed{\sigma^2 = \theta^2 = 9}$.