Math 511 HW # 8 key 3-26-01

P151-2

§ 3.4

3.4-5 $p = 0.2, \quad q = 0.8$

$P = (.8)^3(.2) = 0.1024$

3.4-7 (a) $P(X \geq 20) = P(X > 19) = (.96)^{19} = .4604$

(b) $P(X \leq 20) = 1 - P(X > 20) = 1 - (.96)^{20} = .5580$

(c) $P(X = 20) = \binom{19}{0}(.04)^1(.96)^{19} = .0184$

3.4-10 (a) Negative binomial with $r = 10$, $p = 0.6$, $q = 0.4$;

$\mu = \frac{10}{0.60} = 16.667$

$\sigma^2 = \frac{10(0.40)}{(0.60)^2} = 11.111$

$\sigma = 3.333$

(b) $P(X = 16) = \binom{15}{9}(.6)^{10}(.4)^6 = .1240$

3.4-11 (a) $x = 2$, $f(x) = \frac{1}{2} \cdot \frac{1}{2} * 2$

$x = 3$, $f(x) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} * 2$

$\vdots$

$f(x) = \left(\frac{1}{2}\right)^{x-1} \qquad (x = 2, 3, 4, \cdots)$

(b) $\mu = \sum_{x=2}^{\infty} x\left(\frac{1}{2}\right)^{x-1} = \sum_{x=1}^{\infty} x\left(\frac{1}{2}\right)^{x-1} - 1 = \frac{1}{(1-\frac{1}{2})^2} - 1 = 3$

$\sigma^2 = \sum_{x=2}^{\infty} x^2\left(\frac{1}{2}\right)^{x-1} - \mu^2 = 11 - 9 = 2$

3.4-11 (c) (i) $P(X\le 3) = \frac{1}{2} + \left(\frac{1}{2}\right)^2 = \frac{3}{4}$

(ii) $P(X \ge 5) = 1 - P(X\le 4) = 1 - \left(\frac{1}{2} + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3\right) = \frac{1}{8}$

(iii) $P(X=3) = \left(\frac{1}{2}\right)^{3-1} = \frac{1}{4}$

Prove $E[X^2] = M''(0) = r(r+q)/p^2$

$$M(t) = \sum_{x=r}^{\infty} e^{tx} \binom{x-1}{r-1} p^r (1-p)^{x-r} = \frac{(pe^t)^r}{[1-(1-p)e^t]^r}$$

$$M'(t) = (pe^t)^r(-r)[1-(1-p)e^t]^{-r-1}(-(1-p)e^t) + r(pe^t)^{r-1}pe^t[1-(1-p)e^t]^{-r}$$

$$= r\left(\frac{pe^t}{1-qe^t}\right)^r \frac{1}{(1-qe^t)}$$

$$M''(0) = r^2\left(\frac{p}{1-q}\right)^{r-1} \cdot \frac{p}{(1-q)^2} \cdot \frac{1}{1-q} + r\left(\frac{p}{1-q}\right)^r \frac{q}{(1-q)^2}$$

$$= \frac{r^2}{p^2} + \frac{rq}{p^2}$$

$$= r(r+q)/p^2$$

§3.5

$\lambda = 4$

3.5-1 (a) $P(2\le X\le 5) = P(X\le 5) - P(X\le 1) = \sum_{x=0}^{5} \frac{4^x e^{-4}}{x!} - \sum_{x=0}^{1} \frac{4^x e^{-4}}{x!}$

$= 0.785 - 0.092 = 0.693$

(b) $P(X\ge 3) = 1 - P(X\le 2) = 1 - \sum_{x=0}^{2} \frac{4^x e^{-4}}{x!} = 1 - 0.238 = 0.762$

(c) $P(X\le 3) = \sum_{x=0}^{3} \frac{4^x e^{-4}}{x!} = 0.433$

3.5-3 $\lambda = 11$

$$P(X>10) = 1 - P(X \le 10) = 1 - \sum_{x=0}^{10} \frac{11^x e^{-11}}{x!} = 1 - .460 = .540$$

3.5-5 $\lambda = \frac{225}{150} = 1.5$

$$P(X \le 1) = \sum_{x=0}^{1} \frac{(1.5)^x e^{-1.5}}{x!} = .558$$

3.5-9 $\lambda = 10$

(a) $P(3 \le X \le 7) = P(X \le 7) - P(X \le 2)$

$= .220 - .003 = .217$

(b) $P(X \geq 5) = 1 - P(X \le 4) = 1 - .029 = 0.971$