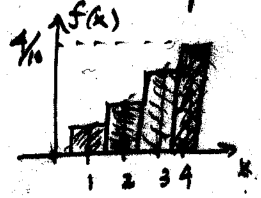
Math / Stat 511 — HW #5, Key — Feb. 15, 2001

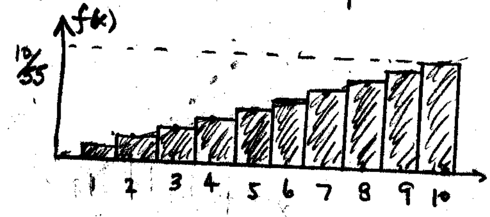
§3.1

#3 (a) If $f(x) = x/c$, $x = 1,2,3,4$, then f will be a pmf when $\sum_{j=1}^{4} j/c = 1$ or when $\boxed{c = 10}$.



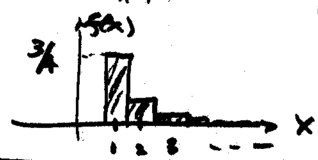
(b) If $f(x) = c \cdot x$, $x = 1,2,\dots,10$, then f is a pmf $\Leftrightarrow$

$$c = \frac{1}{\sum_{j=1}^{10} j} = \frac{1}{\frac{(10)(11)}{2}} = \boxed{\frac{1}{55}}.$$

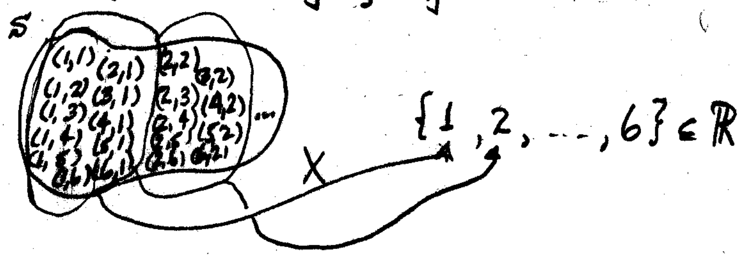


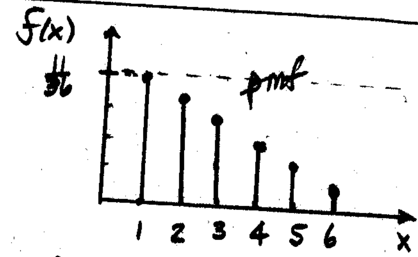
(c) If $f(x) = c\,\frac{1}{4^x}$, $x = 1,2,3,\dots$, then $\sum_{x=1}^{\infty} f(x) = c \sum_{x=1}^{\infty} \frac{1}{4^x} = c \cdot \frac{\frac{1}{4}}{1-\frac{1}{4}}$

$= \frac{c}{3}$ must equal 1. Therefore $\boxed{c = 3}$.



#7 (a) $X((i,j)) = \min(i,j)$, $1 \le i,j \le 6$.





x	$f_X(x)$
1	11/36
2	9/36
3	7/36
4	5/36
5	3/36
6	1/36

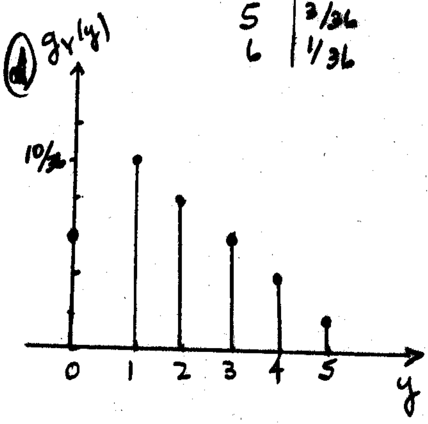
or $f_X(x) = \frac{13-2x}{36}$, $x = 1,2,\dots,6$

(b) Do straightforward drawing from plot in (a).

(c) Let $Y((i,j)) = |i-j|$, $1 \le i,j \le 6$, then

y	$g_Y(y)$
0	6/36
1	10/36
2	8/36
3	6/36
4	4/36
5	2/36

(d)



§ 3.1

#11 $N_1 = 5$ (bad bulbs), $N_2 = 95$ (good bulbs), sample size $n = 10$

The appropriate model for the random variable X = # of defectives in the sample is the hypergeometric distribution.

$$P(\text{at least one defective}) = P(X \geq 1) = 1 - P(X=0)$$

$$= 1 - \frac{\binom{5}{0}\binom{95}{10}}{\binom{100}{10}} \doteq 1 - .5838 \doteq \boxed{.4162}$$

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§3.3

#3 (a) $n=6$ independent Bernoulli trials, probability of success $p = \frac{1}{5}$

$$P((C,I,I,C,I,I)) = \left(\tfrac{1}{5}\right)^2\left(\tfrac{4}{5}\right)^4 \doteq \boxed{.0164}$$

(b) $P(2 \text{ successes}) = P(X=2) = \binom{6}{2}\left(\tfrac{1}{5}\right)^2\left(\tfrac{4}{5}\right)^4 = \boxed{.2458}$

where X has a binomial distribution.

#5 (c) $n = 25$ independent Bernoulli trials, probability of success $p = .7$ (here success means the person selected believes IRS abuses its power.)

$$P(X=12) = \binom{25}{12}(.7)^{12}(.3)^{13} = \boxed{.0115}$$, X binomial distribution

(a) $P(X \geq 13) = P(F \leq 12) = \boxed{.9825}$

F = # of failures has a binomial distribution with $n=25$ & probability parameter $= .3$