Math/Stat 511

Key Homework #4

§2.4

#3 (a) $P(A\cap B) = P(A)\cdot P(B) = \frac{1}{4}\frac{2}{3}$ by independence.

(b) $P(A\cap B') = P(A)\cdot P(B')$
$= \frac{1}{4}\frac{1}{3}$ by independence & Theorem.

(c) $P(A'\cap B') = P(A')\cdot P(B') = \frac{3}{4}\cdot\frac{1}{3}$ by theorem and independence of A & B,

(d) $P((A\cup B)') = P(A'\cap B') = \ldots$ by de Morgan's law

(e) $P(A'\cap B) = P(A')P(B) = \frac{3}{4}\cdot\frac{2}{3}$.

#5
$$\left.\begin{aligned} 0.9 = P(A\cup B) &= P(A)+P(B)-P(A\cap B) \\ &= 0.8+0.5-P(A\cap B)\end{aligned}\right\} \Rightarrow P(A\cap B) = .4$$
$P(A) = .8$, $P(B) = .5$

so it is true that $P(A\cap B) = P(A)P(B)$ & $\therefore$ A & B are independent

#9 (a) $P(A\cap B\cap C) = (.5)(.8)(.9)$, (b) $P\left(\begin{array}{c}\text{exactly 2 of the}\\ \text{3 events occur}\end{array}\right) = P(A\cap B\cap C') +$
$P(A\cap C\cap B') + P(B\cap C\cap A')$ since these are mutually exclusive events.
By independence & the proposition proved

$P(A\cap B\cap C') = P(A)P(B)P(C') = (.5)(.8)(.1)$
$P(A\cap C\cap B') = (.5)(.9)(.2)$
$P(B\cap C\cap A') = (.8)(.9)(.5)$.

#11 (a) $A\cap B = \phi \Rightarrow P(A\cap B) = 0$. For A & B to be independent then one of their probabilities is zero, i.e. one of them is ϕ.

(b) $A\subset B$, then $A\cap B = A \Rightarrow$ & $P(A\cap B) = P(A)\cdot P(B)$, then $P(A) = P(A)P(B)$ so either $A = \phi$ or $B = S$.

#12 (a)(b)(c) $\boxed{\left(\frac{1}{2}\right)^5}$ (d) $\boxed{\binom{5}{3}\left(\frac{1}{2}\right)^5}$

#16 (a) with replacement: Probability (you win) $= \sum_{j=0}^{\infty} \text{Prob}\left(\begin{array}{c}\text{you win on}\\ 2j+1 \text{ draw}\end{array}\right)$
↑ mutually exclusive events

$= \frac{1}{5} + \frac{4}{5}\frac{4}{5}\frac{1}{5} + \left(\frac{4}{5}\right)^4\frac{1}{5} + \cdots$
↑ conditional probabilities

$= \frac{1}{5}\left(\frac{1}{1-16/25}\right) = \boxed{5/9}$
↑ geometric series

#16 (b) without replacement. Again use mutually exclusive events: {you win} = {you win on 1^{st} draw} ∪ {you win on 3^{rd} draw} ∪ {you win on 5^{th} draw}

$$P(\text{you win}) = \frac{1}{5} + \frac{4}{5}\frac{3}{4}\frac{1}{3} + \frac{4}{5}\frac{3}{4}\frac{2}{3}\frac{1}{2}\cdot 1 = \boxed{3/5}$$

---

§2.5 #1 $B_j :=$ event bowl #j selected $(1 \le j \le 4)$. $P(B_1) = \frac{1}{2}$, $P(B_3) = \frac{1}{8}$, $P(B_2) = \frac{1}{4}$, $P(B_4) = \frac{1}{8}$

Note that $\bigcup_{j=1}^{4} B_j = S$.

(a) $W :=$ event of drawing a white chip

$$P(W) = \sum_{j=1}^{4} P(W|B_j)\cdot P(B_j) = \left(1\cdot\tfrac{1}{2}\right) + \left(0\cdot\tfrac{1}{4}\right) + \left(\tfrac{2}{4}\cdot\tfrac{1}{8}\right) + \left(\tfrac{3}{4}\cdot\tfrac{1}{8}\right) = \boxed{\tfrac{21}{32}}$$

know the 'priors'

(b) Compute the posterior conditional probability

$$P(B_1|W) = \frac{P(W|B_1)\cdot P(B_1)}{P(W)} = \frac{1\cdot\frac{1}{2}}{\left(\frac{21}{32}\right)} = \boxed{\tfrac{16}{21}}$$

#5

8 Y 8 W 8 P	6 Y 6 W 12 P
A	B

Assume $P(A) = P(B) = \frac{1}{2}$, where

A = event package A selected

B = — — — B — —

(a) $$P(B\,|\,3\text{ purples}) = \frac{P(E/B)\cdot P(B)}{P(E/A)\cdot P(A) + P(E/B)\cdot P(B)} = \boxed{\tfrac{110}{138}}$$

E = event all 3 purple

1^{st} compute the 'priors' $P(E|A) = \dfrac{\binom{8}{3}\binom{16}{0}}{\binom{24}{3}}$, $P(E|B) = \dfrac{\binom{12}{3}\binom{12}{0}}{\binom{24}{3}}$

(b) Now set D = event of 1 yellow, 1 white, & 1 purple

Computing the priors $P(D|A) = \dfrac{\binom{8}{1}\binom{8}{1}\binom{8}{1}}{\binom{24}{3}}$, $P(D|B) = \dfrac{\binom{6}{1}\binom{6}{1}\binom{12}{1}}{\binom{24}{3}}$

The posterior estimate required is

$$P(A|D) = \frac{P(D|A)\,P(A)}{P(D|A)P(A) + P(D|B)\cdot P(B)} = \boxed{\tfrac{512}{944}}$$