

§2.4

- #3 (a) $P(A \cap B) = P(A) \cdot P(B) = \frac{1}{4} \cdot \frac{2}{3}$ by independence.
 (b) $P(A \cap B') = P(A) \cdot P(B')$
 $= \frac{1}{4} \cdot \frac{1}{3}$ by independence & Theorem.
 (c) $P(A' \cap B') = P(A') \cdot P(B') = \frac{3}{4} \cdot \frac{1}{3}$ by theorem and independence of A & B .
 (d) $P(A \cup B') = P(A' \cap B') = \dots$
 by de Morgan's Law
 (e) $P(A' \cap B) = P(A') P(B) = \frac{3}{4} \cdot \frac{2}{3}$.

#5 $0.9 = P(A \cup B) = P(A) + P(B) - P(A \cap B)$
 $= 0.8 + 0.5 - P(A \cap B) \Rightarrow P(A \cap B) = .4$
 $P(A) = .8, P(B) = .5$

so it is true that $P(A \cap B) = P(A)P(B)$ & $\therefore A$ & B are independent

#9 (a) $P(A \cap B \cap C) = (.5)(.8)(.9)$, (b) $P(\text{exactly 2 of the 3 events occur}) = P(A \cap B \cap C') +$

$P(A \cap C \cap B') + P(B \cap C \cap A')$ since these are mutually exclusive events.

By independence & the proposition proved

$P(A \cap B \cap C') = P(A)P(B)P(C') = (.5)(.8)(.1)$

$P(A \cap C \cap B') = (.5)(.9)(.2)$

$P(B \cap C \cap A') = (.8)(.9)(.5)$.

#11 (a) $A \cap B = \phi \Rightarrow P(A \cap B) = 0$. For A & B to be independent then one of their probabilities is zero, i.e. one of them is ϕ .

(b) $A \subset B$, then $A \cap B = A \Rightarrow P(A \cap B) = P(A) \cdot P(B)$, then $P(A) = P(A)P(B)$
 so either $A = \phi$ or $B = S$.

#12 (a) (b) (a) $\left(\frac{1}{2}\right)^5$ (d) $\left(\frac{5}{3}\right)\left(\frac{1}{2}\right)^5$

#16 (a) with replacement: Probability (you win) = $\sum_{j=0}^{\infty} \text{Pr}(\text{you win on } 2j+1 \text{ draw})$
 \uparrow mutually exclusive events

$= \frac{1}{5} + \frac{4}{5} \cdot \frac{4}{5} \cdot \frac{1}{5} + \left(\frac{4}{5}\right)^4 \cdot \frac{1}{5} + \dots$
 \uparrow conditional probabilities
 $= \frac{1}{5} \left(\frac{1}{1 - \frac{16}{25}} \right) = \boxed{\frac{5}{9}}$
 \uparrow geometric series

#16 (b) without replacement Again use mutually exclusive events: {you win} = {you win on 1st draw} ∪ {you win on 3rd draw} ∪ {you win on 5th draw}

$$P(\text{you win}) = \frac{1}{5} + \frac{4}{5} \cdot \frac{3}{4} \cdot \frac{1}{3} + \frac{4}{5} \cdot \frac{3}{4} \cdot \frac{2}{3} \cdot \frac{1}{2} \cdot 1 = \boxed{\frac{3}{5}}$$

§2.5
ind

#1

B_j = event bowl #j selected ($1 \leq j \leq 4$)

$P(B_1) = \frac{1}{2}, P(B_3) = \frac{1}{8}$

Note that $\bigcup_{j=1}^4 B_j = S$.

$P(B_2) = \frac{1}{4}, P(B_4) = \frac{1}{8}$

(a) W = event of drawing a white chip

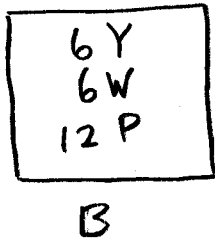
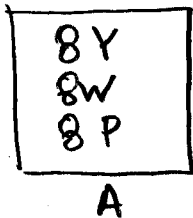
$$P(W) = \sum_{j=1}^4 P(W|B_j) \cdot P(B_j) = (1 \cdot \frac{1}{2}) + (0 \cdot \frac{1}{4}) + (\frac{2}{4} \cdot \frac{1}{8}) + (\frac{3}{4} \cdot \frac{1}{8}) = \boxed{\frac{21}{32}}$$

↑ know the 'priors'

(b) Compute the posterior conditional probability

$$P(B_1|W) = \frac{P(W|B_1) \cdot P(B_1)}{P(W)} = \frac{1 \cdot \frac{1}{2}}{(\frac{21}{32})} = \boxed{\frac{16}{21}}$$

#5



Assume $P(A) = P(B) = \frac{1}{2}$, where

A = event package A selected

B = ----- B -----

(a) $P(B|3 \text{ purple}) = \frac{P(E|B) \cdot P(B)}{P(E|A) \cdot P(A) + P(E|B) \cdot P(B)} = \boxed{\frac{110}{138}}$

E = event all 3 purple

1st compute the 'priors'

$P(E|A) = \frac{\binom{8}{3} \binom{16}{0}}{\binom{24}{3}}, P(E|B) = \frac{\binom{12}{3} \binom{12}{0}}{\binom{24}{3}}$

(b) Now set D = event of 1 yellow, 1 white, 2 purple

Computing the priors $P(D|A) = \frac{(8)(8)(8)}{\binom{24}{3}}, P(D|B) = \frac{(6)(6)(12)}{\binom{24}{3}}$

The posterior estimate required is

$$P(A|D) = \frac{P(D|A) P(A)}{P(D|A) P(A) + P(D|B) \cdot P(B)} = \boxed{\frac{512}{944}}$$