Solutions homework 8.

Problem 1 = Problem 38-2. Solution: No, take $G_1 = (0,1) \cap (1,2)$ and $G_2 = (0,2)$.

Problem 2 = Problem 38-3. Solution: Assume $G \neq \emptyset$. Then there exists a nonempty open interval $I \subset G$. Them $0 < m(I) \leq m(G) = 0$ is a contradiction. Hence $G = \emptyset$.

Problem 3 = Problem 38-5. Solution: From $0 \leq \mu_*(A) \leq \mu^*(A) = 0$ it follows that $\mu_*(A) = \mu^*(A)$, so A is measurable.

Problem 4 = Problem 38-9. Solution: If $B \subset G$ with G open, then also $A \subset G$. This implies that $\mu^*(A) \leq \mu^*(B)$. Now $A \subset B$ implies that $[0.1] \setminus B \subset [0,1] \setminus A$, so by the above we have $\mu^*([0,1] \setminus B) \leq \mu^*([0,1] \setminus A)$. Hence $\mu_*(A) \leq \mu_*(B)$.

Problem 5 = Problem 38-13. Solution: $\mu_*(A) - \mu_*([0,1] \setminus A) = 1 - \mu^*([0,1] \setminus A) - (1 - \mu^*(A)) = \mu^*(A) - \mu^*([0,1] \setminus A)$.