Solutions homework 13.

(1) **Problem 18-15** If A is closed, then $A = \overline{A}$, so we have that the boundary $B(A) = A \cap \overline{A^c}$. This implies that every $x \in B(A)$ is a limit point of A^c, since if $x \in B(A)$, then $x \in A$ and $x \in \overline{A^c}$. Let $x \in B(A)$ and I an open interval with $x \in I$. Then obviously $I \cap A \neq \emptyset$, as $x \in A \cap I$, but also $I \cap A^c \neq \emptyset$ as x is a limit point of A^c. Let now I denote any open interval. If $I \cap A = \emptyset$, then take $J = I$ and $J \cap B(A) \subset I \cap A = \emptyset$ implies that $J \cap B(A) = \emptyset$. Assume therefore that $A \cap I \neq \emptyset$. By the remarks made above there exists $y \in I \cap A^c$. Then y is not a limit point of A implies that there exist an open interval I_1 such that $y \in I_1$ and $I_1 \cap A = \emptyset$. Let now $J = I \cap I_1$. Then $J \subset I$ is an open interval and $J \neq \emptyset$ since $y \in J$ and $J \cap A = \emptyset$. Hence also $J \cap B(A) = \emptyset$. Hence $B(A)$ is nowhere dense.

(2) **Problem 19-2a** The answer is yes. For each n let $I_{k,n} = (\frac{1}{k} - \frac{1}{k2^n}, \frac{1}{k} + \frac{1}{k2^n})$ and define $O_n = \cup_{k=1}^{\infty} I_{k,n}$. Then each O_n is open, as it is a union of open sets and one easily checks that $\cap_n O_n = \{\frac{1}{k} : k = 1, 2 \cdots\}$.