Solutions for HW 1

Exercise 0.0.1: Solution: Let $A_n = \{\alpha \in A : x_\alpha \ge \frac{1}{n}\}$. Then $\{\alpha \in A : x_\alpha > 0\} = \bigcup_{n=1}^{\infty} A_n$. Hence it suffices to show that each A_n is finite. Denote by S the sum $\Sigma_{\alpha} x_{\alpha}$. Then

$$\sum_{\alpha \in A_n} \frac{1}{n} \le \sum_{\alpha \in A_n} x_\alpha \le S$$

implies that A_n has at most nS elements and we are done.