Solutions for HW 8

Problem 1.4: 1

a) $f(n) = 2n - 1$.

b) $f(n) = n - 1$

d) $f(x) = 5x - 4$.

f) $f(x) = \frac{1}{1+e^{-x}}$ or $f(x) = \frac{1}{\pi}(\arctan x + \frac{\pi}{2})$.

Problem 1.4: 4. Let $\epsilon > 0$. Then $\beta - \epsilon < \beta$ implies that there exists $x_1 \in S$ such that $\beta - \epsilon < x_1 < \beta$ (note $x_1 < \beta$, since $\beta \notin S$). Now $x_1 < \beta$ implies that there exists $x_2 \in S$ such that $x_1 < x_2 < \beta$. Assuming we have found $x_{n-1} \in S$ with $x_{n-1} < \beta$ we can find $x_n \in S$ such that $x_{n-1} < x_n < \beta$. By induction we find the infinite set $\{x_n : n = 1, \cdots\}$ with $\beta - \epsilon < x_n < \beta$.

Problem 1.4: 9. Let A be countable and B be uncountable. If $A \cup B$ is countable, then it follows that B is countable as B is a subset of $A \cup B$. Hence $A \cup B$ is uncountable.