Solutions for HW 7

Problem 1.3: 19. Let p the integer such that $p \leq s+1 < p+1$. Then $s+1 < p+1$ implies that $s < p$. From $p \leq s+1 < t$ we get that $p < t$, so we are done.

Problem 1.3: 20. Let $\epsilon > 0$. Then there exists a rational number r such that $x < r < x + \epsilon$. This r satisfies $0 < |x - r| < \epsilon$.

Problem 1.3: 21. Let $x < y$ be two given real numbers. Then by the Archimedean property there exists a natural number q such that $q(y-x) > 1$, but then also $2^q(y-x) > 1$ (since $2^q > q$). Now find via problem 19 an integer p such that $2^q x < p < 2^q y$. Dividing by 2^q we get the required dyadic rational between x and y.

Problem 1.3: 29. Let $\beta = \sup(-S)$. Then $-x \leq \beta$ for all $x \in S$ implies that $x \geq -\beta$ for all $x \in S$, so $-\beta \leq \inf S$. If $\gamma > -\beta$, then $-\gamma < \beta$. Thus $-\gamma$ is not an upper bound for $-S$, so γ is not a lower bound for S. Hence $-\beta = \inf S$, i.e., $-\sup(-S) = \inf S$.