Solutions for HW 6

Problem 1.3: 8. Assume $\alpha = \sup S = \beta$. Then α is an upper bound, so by the second property of β being the supremum we get that $\beta \leq \alpha$. Similarly from α being the supremum we get that $\alpha \leq \beta$. Thus $\alpha = \beta$.
Problem 1.3: 10. Let $S = (a, b)$. The $a < x < b$ for all $x \in S$ implies that a is a lower bound for S and b is an upperbound for S. Let $a < \gamma < b$. Then put $x = \frac{a+\gamma}{2}$ and $y = \frac{\gamma+b}{2}$. Then $x, y \in S$ and $a < x < \gamma < y < b$, so that γ is neither a lower or upper bound for S. Hence $a = \inf S$ and $b = \sup S$.
Problem 1.3: 11. Let $S = \{x_1, \cdots, x_n\}$. By rearranging the terms we can assume that $x_1 < x_2 < x_3 < \cdots < x_n$. Then $x_1 = \min S$ and thus $\inf S = \min S = x_1$.
Problem 1.3: 13. Note $3x^2 - 10x + 3 = (3x-1)(x-3)$, so $\{x : 3x^2 + 3 < 10x\} = (\frac{1}{3}, 3)$. Therefor $\sup\{x : 3x^2 + 3 < 10x\} = 3$ by problem 10.
Problem 1.3: 26. Let $\alpha = \sup S$ and $\beta = \sup A$. Then β is an upper bound for A and thus for S. As α is the least upper bound of S it follows that $\alpha \leq \beta$, i.e., $\sup S \leq \sup A$. Similarly one can show that $\inf A \leq \inf S$. It remains to show $\inf S \leq \sup S$. To see this let $x \in S$. Then $\inf S \leq x$ and $x \leq \sup S$, so $\inf S \leq \sup S$.