Solutions for HW 5

Problem 1.2: 24. Using the Cauchy-Schwarz inequality we have

$$(\sum_{k=1}^{n} |a_k|)^2 \leq \sum_{k=1}^{n} |a_k|^2 \sum_{k=1}^{n} 1^2 = n \sum_{k=1}^{n} |a_k|^2.$$

Taking square roots on both sides gives the desired inequality.

Problem 1.3: 2. Note $10\sqrt{x} - x = \sqrt{x}(10 - \sqrt{x}) > 0$ if and only if $x > 0$ and $\sqrt{x} < 10$, i.e. if and only if $0 < x < 100$. Hence 100 is a bound for the given set.

Problem 1.3: 3. Note $x^2 - 25x = x(x - 25) > 0$ if and only if $x < 0$ or $x > 25$. This shows that the set $\{x : x^2 - 25x > 0\} = (-\infty, 0) \cup (25, \infty)$, which is not bounded from below or above.

Problem 1.3: 4. Let $S = \{x_1, \cdots, x_n\}$. Then $M = |x_1| + \cdots + |x_n|$ is a bound for $S$.

Problem 1.3:6. Let $S_1$ be bounded by $M_1$ and $S_2$ be bounded by $M_2$. Then $S_1 \cup S_2$ is bounded by $M_1 + M_2$.