Solutions for HW 3

Problem 1.2: 6. By the triangle inequality $|x-y| = |(x-a)+(a-y)| \leq |x-a| + |a-y| < \epsilon + \epsilon = 2\epsilon$.

Problem 1.2: 8.

a. From $a_1 \leq |a_1|, \cdots, a_n \leq |a_n|$ it follows that

$$a_1 + \cdots + a_n \leq |a_1| + \cdots + |a_n|$$

and from $-a_1 \leq |a_1|, \cdots, -a_n \leq |a_n|$ it follows that

$$-(a_1 + \cdots + a_n) \leq |a_1| + \cdots + |a_n|.$$

Combining these two inequalities we get

$$|a_1 + \cdots + a_n| \leq |a_1| + \cdots + |a_n|,$$

which is the desired inequality.

b.

$$|a_1| = |\sum_{k=1}^{n} a_k - \sum_{k=2}^{n} a_k| \leq |\sum_{k=1}^{n} a_k| + |\sum_{k=2}^{n} a_k| \leq |\sum_{k=1}^{n} a_k| + \sum_{k=2}^{n} |a_k|,$$

which is equivalent to the required inequality.

Problem 1.2: 9. From $|\,|a| - |x|\,| \leq |x-a| < \frac{|a|}{2}$ we conclude that $-\frac{|a|}{2} < |x| - |a| < \frac{|a|}{2}$. Hence $\frac{|a|}{2} < |x| < \frac{3|a|}{2}$.

Problem 1.2: 15.

a. $(-\sqrt{6}, \sqrt{6})$.

b. $[2, \infty)$.

c. $[-2, 1)$

d. $(-\infty, 1]$.

e. $(-2, 0)$.