Solutions for HW 27

Problem 4.1: 4 For $x < 1$ the function f is a polynomial and thus differentiable and $f'(x) = 2x$. For $x > 1$ the function f is a linear function and thus differentiable and $f'(x) = 2$. Remains to check that f is differentiable at $x = 1$. We compute first

$$\lim_{h\to 0+} \frac{f(1+h) - f(1)}{h} = \lim_{h\to 0+} \frac{2h}{h} = 2,$$

and

$$\lim_{h\to 0-} \frac{f(1+h) - f(1)}{h} = \lim_{h\to 0-} \frac{2h + h^2}{h} = 2.$$

Thus $f'(1) = 1$.
Problem 4.1: 5 Note, if you compute $f'(x)$ for $x \neq 0$ and try to take the limt for $x \to$), you do not get anywhere, as these limits do not exist. Therefore we have to use the definition. For $r = 2$ we get

$$\lim_{h\to 0} \frac{f(h) - f(0)}{h} = \lim_{h\to 0} h \cos\left(\frac{1}{h}\right) = 0$$

(by the squeeze theorem), so the derivative exists at 0 and $f'(0) = 0$. On the other hand for $r = 1$ we get

$$\lim_{h\to 0} \frac{f(h) - f(0)}{h} = \lim_{h\to 0} \cos\left(\frac{1}{h}\right),$$

which does not exist.
Problem 4.1: 16 By the product rule we have $(fg)'(2) = 5 \cdot 2 + 4 \dot{-} 3 = -2$, by the quotient rule we have $(\frac{f}{g})'(2) = \frac{11}{2}$ and by the chain rule $(f \circ g)'(2) = f'(g(2))g'(2) = 5 \cdot -3 = -15$.
Problem 4.1: 22 T o find the derivative of $\sin x$ we compute

$$\begin{aligned}\lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h} &= \lim_{h\to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h} \\ &= \lim_{h\to 0} \sin x \frac{\cos h - 1}{h} + \cos x \frac{\sin h}{h} = \cos x.\end{aligned}$$