Solutions for HW 26

Problem 3.5: 8 Let $D = \{x \in [a,b] : f \text{ discontinuous at } x\}$. Then by Theorem 3.33 D is countable. Let now $I \subset [a,b]$ be any interval. Then I is uncountable implies that $I \setminus D$ is non-empty, since otherwise $I \subset D$ and that would imply that I is countable. In fact $I \setminus D$ must be uncountable, since otherwise $I = (I \cap D) \cup (I \setminus D)$ is countable.

Problem 3.5: 9

- **a.** Define

$$f(x) = \begin{cases} 0 & \text{if } x \le 1 \\ 1 & \text{if } 1 < x < 2 \\ 2 & \text{if } 2 \le x < 5 \\ 3 & \text{if } x \ge 5. \end{cases}$$

- **b.** Define $f(x) = 0$ for $x \le 1$ and $f(x) = n$ for $n < x \le n+1$ for $n \ge 1$.
- **c.** Let $f(x) = \lfloor x \rfloor$ the greatest integer function.
- **d.** Let $f(x) = \arctan \lfloor x \rfloor$.
- **e.** Define $f(x) = \frac{1}{2^n}$ for $\frac{1}{n+1} < x \le \frac{1}{n}$ for $n \ge 1$ and $f(x) = 0$ for $x \le 0$ and $f(x) = 1$ for $x > 1$.

Problem 3.6: 6 Let $x_n = 10^{-n}$, Then $\lim f(x_n) = 1$. Let $y_n = 2^{-n}$. Then $\lim f(y_n) = 0$. Hence $\lim_{x \to 0+} f(x)$ does not exist.

Problem 3.6: 14 Let $a = 0$ and $b = 1$. Then $f(0) = 1 < \frac{3}{2} < f(1) = 2$, but there does not exist $0 < x < 1$ such that $f(x) = \frac{3}{2}$, as $f(x) > 2$ on $(0,1)$.