Solutions for HW 25

Problem 3.4: 7 First Solution Let $\epsilon > 0$. Then there exists $\delta > 0$ such that $u, v \in J$ with $|u - v| < \delta$ implies that $|f(u) - f(v)| < \epsilon$. Using this δ we can find $\delta_1 > 0$ such that $x, y \in I$ with $|x - y| < \delta_1$ implies that $|g(x) - g(y)| < \delta$. As $g(x), g(y) \in J$ we can take $u = f(x)$ and $v = g(y)$ in the first paragraph to obtain that $|f(g(x)) - f(g(y))| < \epsilon$ if $x, y \in I$ with $|x - y| < \delta_1$, i.e., $f \circ g$ is uniformly continuous.
Second Solution Let $x_n - y_n \to 0$ in I. Then $g(x_n) - g(y_n) \to 0$ in J. Thus $f(g(x_n)) - f(g(y_n)) \to 0$. Hence $f \circ g$ is uniformly continuous.
Problem 3.6: 3 Note $P(0) = a_0$ and $\lim_{x \to \pm\infty} P(x) = a_n \cdot \infty (= \infty$ if $a_n > 0$ and $-\infty$ if $a_n < 0)$. From this and $a_n a_0 < 0$ it follows now that if $P(0) = a_0 > 0$, then these limits equal $-\infty$ so that there exist $a < 0$ and $b > 0$ with $P(a) < 0$ and $P(b) < 0$. By the intermediate value Theorem P has a zero between a and 0, and between 0 and b, so at least two real zeros. If $P(0) = a_0 < 0$ there exist $a < 0$ and $b > 0$ with $P(a) > 0$ and $P(b) > 0$ and the result follows as above.