Solutions for HW 22

Problem 3.3: 11 Observe $f(x) = (x-1)^2 + 2$, so $f(x) \geq f(1) = 2$ for all x. In particular f has a minimum at $x = 1 \in (0, 3)$. The function f has no maximum as $\sup\{f(x) : x \in (0,3)\} = 6$, but $f(x) < 6$ if $0 < x < 3$.

Problem 3.3: 12 Since $\lim_{x\to 0} \frac{\sin x}{x} = 1$ we get that $\lim_{x\to 0+} g(x) = 0$. Hence 0 is a removable discontinuity for g and g can be extended to a continuous function on $[0, 2\pi]$ by defining $g(0) = 0$. Then by the extreme value theorem g has a maximum and minmum on $[0, 2\pi]$. As $g(x) > 0$ on $(0, 2\pi]$ the function g does not have a minimum on $(0, 2\pi]$, but the maximum must at some point in $(0, 2\pi]$.

Problem 3.3: 17

a. If $Q(x) = x^2$, then $Q(I) = I$.
b. If $Q(x) = x(1-x)$, then $Q(I) = (0, \frac{1}{4}]$.
c. There is no such Q, as it would have to a max and a min in I, which is impossible for a quadratic polynomial.

Problem 3.3:18 Let

$$f(x) = \begin{cases} 0 & \text{if } 0 < x < \frac{1}{4}, \\ 2x - \frac{1}{2} & \text{if } \frac{1}{4} \leq x \leq \frac{3}{4} \\ 1 & \text{if } \frac{3}{4} < x < 1, \end{cases}$$

then $f((0,1)) = [0,1]$. Note we can also write $f(x) = \min(1, \max(0, 2x - \frac{1}{2}))$.