Solutions for HW 20

Problem 3.2: 11

a. Let $g(x) = |x|$. Then g is continuous on $\mathbb{R}$, so by the composition theorem $|f| = g \circ f$ is continuous. A more direct proof follows from $\big| |f(x)| - |f(c)| \big| \leq |f(x) - f(c)|$.

b. Let $g(x) = \sqrt{x}$ for all $x \geq 0$. Then $\sqrt{f} = g \circ f$ is a composition of two continuous function and thus continuous.

Problem 3.2: 14 for f to be continuous on I we need that $x^2 - 20 \neq 0$ on I., i.e., $x \neq \pm\sqrt{20}$. Hence $I = (-\sqrt{20}, \sqrt{20})$.

Problem 3.2: 18 Note first that the numerator $x + x^2$ is a polynomial, thus defines an everywhere continuous function f. Secondly the function g defined by $g(x) = \sin^3(6x)$ is continuous as it is the composition of three continuous functions. Moreover $g(x) \geq 2 - 1 = 1$ implies that g is nowhere zero. Hence $\frac{f}{g}$ is everywhere continuous.

Problem 3.2: 19 For $x \neq 0$ the function $g = h \circ f$, where $h(x) = \sin x$ on $\mathbb{R}$ and $f(x) = \frac{1}{x}$ for $x \neq 0$. As both h and f are continuous we conclude that g is continuous

Problem 3.2:23 Note $x^2 - 1 = (x-1)(x+1)$, so f not defined at $x = \pm 1$. For $x = -1$ the numerator $x^3 - 3x^2 + 2$ equals -2, so the discontinuity at $x = -1$ is not removable. On the other hand $x^3 - 3x^2 + 2 = (x-1)(x^2 - 2x - 2)$ implies that $\lim_{x \to 1} f(x) = -\frac{3}{2}$, so the discontinuity at $x = 1$ is removable for f. For g observe $x^3 - 2x^2 + x = x(x-1)^2$, while $x^2 + 4x - 5 = (x-1)(x+5)$, which shows that the discontinuities at $x = 0$ and $x = 1$ are both not removable.