Solutions for HW 2

Problem 1.1: 7. Let x be rational and y be irrational. Then we prove by contradiction that $x + y$ is irrational. Assume on the contrary that $x + y$ is rational. Then $(x + y) - x = y$ is also rational, as the sum or difference of two rational numbers is again rational. Contradiction. Hence $x + y$ is irrational. (Note: in class I indicated that the sum of two rational numbers is again rational).
Problem 1.1: 9. Let x be irrational and take $y = \frac{1}{x}$. Then by problem 1.1:6 we have that y is also irrational. Now $xy = 1$ is rational.
Problem 1.1:10. Assume that both $\sqrt{2} - x$ and $\sqrt{2} + x$ are rational. Then also their sum $(\sqrt{2} - x) + (\sqrt{2} + x) = 2\sqrt{2}$ is rational and thus also $\frac{1}{2}(2\sqrt{2}) = \sqrt{2}$ is rational. Contradiction. Hence at least one of $\sqrt{2} - x$ or $\sqrt{2} + x$ is irrational.
Problem 1.2: 1.

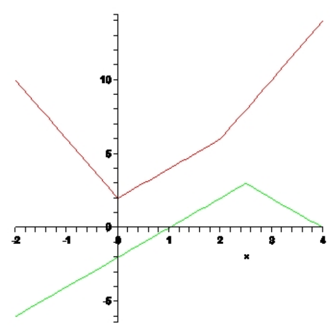

FIGURE 1. Top graph is $3|x| + |x - 2|$. Bottom graph is $3 - |2x - 5|$

Problem 1.2: 3. Note by definition $\sqrt{a}$ is the nonengative solution of $x^2 = a$, so $\sqrt{x^2} = |x|$ as $|x|$ is nonnegative and $|x|^2 = x^2$ (since $(-x)^2 = x^2$).