Solutions for HW 19

Problem 3.1: 32 Note $f(-1+) = \lim_{x\to -1+} -x^2 = -1$. Also $f(0-) = \lim_{x\to 0-} -x^2 = 0, f(0+) = \lim_{x\to 0+} 2 = 2, f(2-) = \lim_{x\to 2-} 2 = 2, f(2+) = \lim_{x\to 2+} x+3 = 5$, and $f(3-) = \lim_{x\to 3-} x+3 = 6$.

Problem 3.1:33 $f(1-) = 3a+1$, $f(1+) = a+b$, so f has a limit at $x = 1$ if $3a+1 = a+b$, or $2a-b = -1$. Similarly $f(2-) = 2a+b = f(2+) = 4b+a$ if and only if $a = 3b$. Solving the two equations for a and b, we find $a = -\frac{3}{5}$ and $b = -\frac{1}{5}$.

Problem 3.2:3 Let $\epsilon > 0$. For $c = 0$ we take $\delta = \epsilon^2$ Then $0 < x < \delta$ implies that $0 < \sqrt{x} < \sqrt{\delta} = \epsilon$, so f continuous at 0. For $c > 0$ we have that $|\sqrt{x} - \sqrt{c}| = |\frac{x-c}{\sqrt{x}+\sqrt{c}}| \leq \frac{1}{\sqrt{c}}|x-c|$. Hence if we choose $\delta = \sqrt{c}\cdot\epsilon$. then it follows from $0 < |x-c| < \delta$ that $|\sqrt{x} - \sqrt{c}| < \frac{1}{\sqrt{c}}\delta = \epsilon$. Hence f is also continuous at any $c > 0$.

Problem 3.2:4 Let $\epsilon = \frac{1}{2}f(c)$. Then there exists $\delta > 0$ such that $0 < |x-c| < \delta$ and $x \in [a,b]$ implies $|f(x) - f(c)| < \epsilon = \frac{1}{2}f(c)$. Note $|f(x) - f(c)| < \epsilon = \frac{1}{2}f(c)$ implies that $f(x) > \frac{1}{2}f(c)$. Hence we can take $m = \frac{1}{2}f(c)$ and $[u,v] = [a,b] \cap [c - \frac{1}{2}\delta, c + \frac{1}{2}\delta]$.

Problem 3.2:7 Let $x \in \mathbb{R}$ be an irrational number. Then there exist $x_n \in \mathbb{Q}$ such that $x_n \to x$. Then $f(x) = \lim f(x_n) = 0$. Hence $f(x) = 0$ for all x.