Solutions for HW 18

Problem 3.1: 4

a. Let $\epsilon > 0$. Then choose $\delta = \frac{\epsilon}{5}$. Then $|(5x - 11) - (-1)| = |5x - 10| = 5|x - 2| < 5\delta = \epsilon$, if $0 < |x - 2| < \delta$.
b. Let $\epsilon > 0$. Observe first that $|(x^2 + x - 1) - 1| = |x + 2||x - 1|$, so to make this small we need to take $|x - 1|$ small enough AND bound $|x + 2|$. Note if $|x - 1| < 1$, then $|x + 2| < 4$. Hence choose $\delta = \min\{1, \frac{\epsilon}{4}\}$. Then $0 < |x - 1| < \delta$ implies that $|(x^2 + x - 1) - 1| = |x + 2||x - 1| < 4\delta \leq \epsilon$.

Problem 3.1:5 Let $\epsilon > 0$. Observe first that $|x^2 - c^2| = |x - c||x + c|$ so to make this small we need to take $|x - c|$ small enough AND bound $|x + c|$. Note if $|x - c| < 1$, then $|x + c| < 1 + 2|c|$. Hence choose $\delta = \min\{1, \frac{\epsilon}{1+2|c|}\}$. Then $0 < |x - c| < \delta$ implies that $|x^2 - c^2| = |x - c||x + c| < \delta(1 + 2|c|) \leq \epsilon$.

Problem 3.1:10 Take $x_n = \frac{1}{2\pi n}$ and $y_n = \frac{1}{\frac{\pi}{2} + 2\pi n}$. Then $x_n \to 0$ and $y_n \to 0$, while $\sin\frac{1}{x_n} = 0$ for all $n \geq 1$ and $\sin\frac{1}{y_n} = 1$ for all $n \geq 1$. Hence by Theorem 3.2 b) the limit does not exist.

Problem 3.1:13 Let $c \in \mathbb{R}$. Then there exist $x_n \in \mathbb{Q}$ such that $x_n \to c$ and there exist $y_n \in \mathbb{R} \setminus \mathbb{Q}$ such that $y_n \to c$ (to see this observe that the intervals $(c - \frac{1}{n}, c)$ contain a rational x_n and an irrational y_n). Now $\lim f(x_n) = 1$ and $\lim f(y_n) = 0$, so $\lim_{x \to c} f(x)$ does not exist and thus f is not continuous at c.