Solutions for HW 12

Problem 2.1:11

a. Let $\epsilon > 0$. Then there exists N such that $|x_n - L| < \epsilon$ for all $n \geq N$. Now $|\,|x_n| - |L|\,| \leq |x_n - L| < \epsilon$ for all $n \geq N$. Hence $|x_n| \to |L|$.

b. Take $x_n = (-1)^n$.

c. Let $\epsilon > 0$. Then there exists N such that $|\,|x_n| - 0| < \epsilon$ for all $n \geq N$, i.e. $|x_n| < \epsilon$ for all $n \geq N$. Thus $|x_n - 0| < \epsilon$ for all $n \geq N$.

Problem 2.1:15

a. $x_n = 3 + \frac{1}{n}$.

b. $y_n = 3 + \frac{1}{n}$ for n even and $y_n = 3$ for n odd.

c. $z_n = 3 - \frac{1}{n}$.

Problem 2.1: 16 Suppose the range of the sequence $\{x_n\}$ is the set $\{a_1, \cdots, a_n\}$. If the sequence converges, then its limit must be one of $\{a_1, \cdots, a_n\}$ and $x_n = a_k$ for some k and all $n \geq N$.