Solutions for HW 1

Problem 1.1: 2. Let $x, y \in \mathbb{Q}$. Then $x = \frac{p}{q}$ and $y = \frac{m}{n}$ with $q, m \neq 0$. Now $\frac{x+y}{2}$ is between x and y (make a picture, it is the midpoint between x and y) and $\frac{x+y}{2} = \frac{pn+mq}{2mq}$ is a rational number. Note a formal proof that the midpoint is between x and y goes as follows: We can assume $x < y$ (otherwise interchange the role of x and y). Then $2x = x + x < x + y$ implies $x < \frac{x+y}{2}$. Similarly $x + y < y + y = 2y$ proves that $\frac{x+y}{2} < y$.

Problem 1.1: 3

a. $x = ..2962962\cdots$

b. $x = .19047619\cdots$

Problem 1.1:4

a. $x = \frac{357}{999}$

b. $x = \frac{18}{55}$

Problem 1.1: 6. Let x be an irrational number and assume that $\frac{1}{x}$ is rational. Then there exist $p, q \in \mathbb{Z}$ such that $\frac{1}{x} = \frac{p}{q}$. Then $x = \frac{q}{p}$, which implies that x is rational. Contradiction. Thus $\frac{1}{x}$ is irrational. Note x irrational implies that $x \neq 0$, so that $\frac{1}{x}$ makes sense.