Homework 9

(1) Describe the range γ^* and the way in which it is traversed for the following curves γ:

    a. $\gamma(t) = 1 + ie^{it}$ $(0 \leq t \leq \pi)$.

    b. $\gamma(t) = e^{it}$ $(-2\pi \leq t \leq \pi)$.

    c.

$$\gamma(t) = \begin{cases} e^{it}, & 0 \leq t \leq \pi \\ e^{-it}, & \pi \leq t \leq 2\pi. \end{cases}$$

(2) Evaluate $\int_\gamma f(z)\, dz$ when

    a. $f(z) = |z|^2$ and γ is the line segment $[-1+i, 1+i]$ starting at $-1+i$ and ending at $1+i$.

    b. $f(z) = \operatorname{Re} z$ and $\gamma(t) = t + it^2$ for $0 \leq t \leq 1$

    c. $f(z) = \frac{1}{z}$ and γ is the join of the line segments $[1 - i, 1 + i]$, $[1 + i, -1 + i]$, and $[-1 + i, -1 - i]$, starting at $1 - i$ and traversing the curve once (see figure 1).
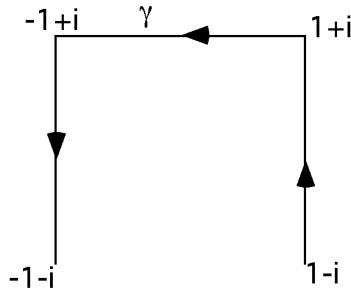

FIGURE 1. γ