Homework 9

(1) Let $E \subset [0,1] \times [0,1]$ be a measurable set. Assume that $m(E_x) \leq \frac{1}{2}$ for almost every $x \in [0,1]$. Prove that $m(\{y \in [0,1] : m(E^y) = 1\}) \leq \frac{1}{2}$.

(2) Let $f \in L^1(\mathbb{R})$ and define for $h > 0$

$$\phi_h(x) = \frac{1}{2h} \int_{x-h}^{x+h} f(t)\, dt.$$

Prove that ϕ_h is integrable and $\|\phi_h\|_1 \leq \|f\|_1$.