Homework 9.

(1) Describe the range γ^* and the way in which it is traversed for the following curves γ:

a. $\gamma(t) = 1 + ie^{it} \quad (0 \le t \le \pi)$.

b. $\gamma(t) = e^{it} \quad (-2\pi \le t \le \pi)$.

c.

$$\gamma(t) = \begin{cases} e^{it}, & 0 \le t \le \pi \\ e^{-it}, & \pi \le t \le 2\pi. \end{cases}$$

(2) Evaluate $\int_\gamma f(z)\,dz$ when

a. $f(z) = |z|^2$ and γ is the line segment $[-1+i, 1+i]$ starting at $-1+i$ and ending at $1+i$.

b. $f(z) = \mathrm{Re}\ z$ and $\gamma(t) = t + it^2$ for $0 \le t \le 1$

c. $f(z) = \frac{1}{z}$ and γ is the join of the line segments $[1-i, 1+i]$, $[1+i, -1+i]$,and $[-1+i, -1-i]$, starting at $1-i$ and traversing the curve once (see figure 1).

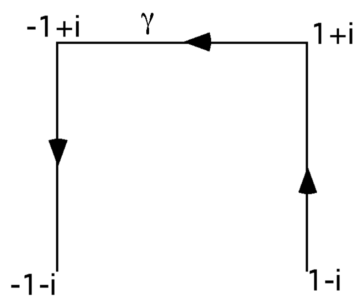

FIGURE 1. γ

(3) Compute

$$\int_0^{2\pi} e^{\cos t}[\cos(\sin t + t)]\,dt$$

and

$$\int_0^{2\pi} e^{\cos t}[\sin(\sin t + t)]\,dt$$

by computing $\int_\gamma e^z\,dz$ using $\int_\gamma f(z)\,dz = \int_0^{2\pi} F(\gamma(t))\gamma'(t)\,dt$, where $\gamma(t) = e^{it}$ with $0 \le t \le 2\pi$.

(4) Evaluate (without parametrizing) $\int_\gamma \frac{1}{1+z^2}\,dz$ for

a. $\gamma(t) = 1 + e^{it}$ $(0 \le t \le 2\pi)$.

b. $\gamma(t) = -i + e^{it}$ $(0 \le t \le 2\pi)$.

c. $\gamma(t) = 2e^{it}$ $(0 \le t \le 2\pi)$.

d. $\gamma(t) = 3i + 3e^{it}$ $(0 \le t \le 2\pi)$.

(5) Let $\alpha \in \mathbb{C}$ with $|\alpha| \ne 1$.

a. Show

$$\int_0^{2\pi} \frac{d\theta}{1 - 2\alpha\cos\theta + \alpha^2} = \frac{i}{\alpha}\int_\gamma \frac{1}{(z-\alpha)(z-\frac{1}{\alpha})}\,dz$$

where $\gamma(t) = e^{it}$ with $0 \le t \le 2\pi$.

b. Compute the integral in part a. (hint use partial fractions).