Homework 9.

(1) Let $A \subset [0,1]$ be Lebesgue measurable. Prove that for $\epsilon > 0$ there exist an open set $O \supset A$ and a closed set $F \subset A$ such that $\mu(O \setminus F) < \epsilon$. (Hint: Use Theorem 38.11 of the text.)

(2) Let $A_1, A_2 \subset [0,1]$ be measurable with $\mu(A_1) = 1$. Prove that $\mu(A_1 \cap A_2) = \mu(A_2)$.

(3) Problem 38.10 of the text

(4) Prove that $\mu^*(\mathbb{Q} \cap [0,1]) = 0$.

(5) Prove that $[0,1] \setminus \mathbb{Q}$ is measurable and $\mu([0,1] \setminus \mathbb{Q}) = 1$.