Homework 8.

(1) Let $G \subset \mathbb{C}$ be an open and connected set and let $f : G \to \mathbb{C}$ be a analytic function such that $f'(z) = 0$ for all $z \in G$. Prove that f is constant on G. (Hint: let $S = \{z \in G : f(z) = f(z_0)\}$ for some fixed $z_0 \in G$ and show that S is open and closed. You can use from undergraduate analysis that if a real valued differentiable function has zero derivative on an interval, then that function is constant.)
(2) Let f be analytic on the unit disk $B(0;1)$.
   a. Prove that if $\operatorname{Re} f$ is constant on $B(0;1)$, then f is constant.
   b. Prove that if e^f is constant on $B(0;1)$, then f is constant.
(3) Find all solutions of
   a. $e^z = -i$.
   b. $\sin z = 0$.
   c. $\operatorname{Ln} z = 1 + i$.
(4) Let $G \subset \mathbb{C}$ be open and let f be analytic on G. Let $G^* = \{z : \bar{z} \in G\}$ and define $f^*(z) = \overline{f(\bar{z})}$ for all $z \in G^*$. Prove that $f^*$is analytic on G^* and express $f^*(z)'$ in terms of f'.
(5) (Quals '04) Let $G \subset \mathbb{C}$ be an open and connected set and let $f : G \to \mathbb{C}$ be a analytic function such that $|f(z)| = C$ for all $z \in G$. Prove that f is constant on G.