Homework 8, Additional Problem.

(1) **a.** Let $f \in L^r$ for some $r < \infty$. Prove that

$$\|f\|_p \leq \|f\|_r^{\frac{r}{p}} \|f\|_\infty^{1-\frac{r}{p}}$$

for all $r < p < \infty$.

b. Asume $f \in L^r$ for some $r < \infty$. Prove

$$\lim_{p\to\infty} \|f\|_p = \|f\|_\infty.$$

(Hint: Use **a.**to get an upper bound for $\overline{\lim}_{p\to\infty} \|f\|_p$ and then use that for $0 \leq t < \|f\|_\infty$ the set $A = \{x : |f(x)| \geq t\}$ has positive measure)