Homework 8

1. Expand $\frac{1}{(1-z)^2}$ and $\frac{1}{(1-z)^3}$ in a power series around $z = 0$.

2. Expand $\frac{2z+3}{z+1}$ in a power series in powers of $z - 1$ (i.e., around $z = 1$). What is the radius of convergence of the series.

3. If $\sum_{n=0}^{\infty} a_n z^n$ has radius of convergence R, what is the radius of convergence of $\sum_{n=0}^{\infty} a_n (2z)^n$ and of $\sum_{n=0}^{\infty} a_n^2 z^n$.

4. If $\sum_{n=0}^{\infty} a_n z^n$ has radius of convergence R_1 and $\sum_{n=0}^{\infty} b_n z^n$ has radius of convergence R_2, prove that $\sum_{n=0}^{\infty} a_n b_n z^n$ has radius of convergence R where $R \geq R_1 R_2$.

5. Show that

$$\mathrm{Log}(1 - z) = \sum_{n=1}^{\infty} -\frac{z^n}{n}$$

for $|z| < 1$, by proving that both sides are holomorphic on $|z| < 1$, agree at $z = 0$, and have the same derivative on $|z| < 1$.