Homework 7

1. Prove that if $z = x + iy$ and $f(z) = \sqrt{(|xy|)}$, then the real part and imaginary part of f satisfy the Cauchy-Riemann equations at $z = 0$, but f is not differentiable at $z = 0$.

2. Find power series expansions around $z = 0$ and indicate the radius of convergence for

   a. $f(z) = \frac{1}{1+z^3}$

   b. $f(z) = \frac{1}{(z+1)(z+2)}$

3. Let $G \subset \mathbb{C}$ be open and let $f \in H(G)$. Let $G^* = \{z : \bar{z} \in G\}$ and define $f^*(z) = \overline{f(\bar{z})}$ for all $z \in G^*$. Prove that $f^* \in H(G^*)$ and express $f^*(z)'$ in terms of f'.

4. Let $a_n > 0$ in $\mathbb{R}$ and assume that $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = L$, where $L \in \mathbb{R} \cup \{\infty\}$. Prove that $\limsup \sqrt[n]{a_n} = L$.