Homework 6, Additional Problems.

(1) Let $f_n : \mathbb{R} \to \mathbb{R}$ be defined by $f_n(x) = x^{2n}\,(1 + x^{2n})^{-1}$. Prove that the pointwise limit $\lim_{n\to\infty} f_n(x)$ exists for all $x \in \mathbb{R}$, but that f is discontinuous at $x = \pm 1$.

(2) Define $f_n(x) = nxe^{-nx^2}$ for $x \in \mathbb{R}$. Prove that $f_n(x) \to 0$ pointwise on $\mathbb{R}$, but not uniform on any interval containing 0. (Hint for the second part: consider the points $x_n = \frac{1}{\sqrt{n}}$.)

(3) **a.** Prove Exercise 14.3.8.
b. Show by means of an example that **a.** can fail if we drop the boundedness condition.