Homework 6, due October 12

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5. Let (X, d) be a metric space and $A \subset X$ a non-empty subset of X. Define the distance from x to A by

$$d(x, A) = \inf\{d(x, y) : y \in A\}$$

a. Show $x \in \bar{A}$ if and only if $d(x, A) = 0$.

b. Show

$$|d(x, A) - d(y, A)| \leq d(x, y).$$

c. Show that $f_A(x) = d(x, A)$ is uniformly continuous on X.

d. Let now A and B be disjoint closed subsets of X. Prove that there exists a continuous function f on X such that $f(x) = 0$ on A and $f(x) = 1$ on B. (Hint: Consider the function $f(x) = \frac{f_A(x)}{f_A(x)+f_B(x)}$.)