Homework 6, Additional Problem.

(1) **a.** Let $a_n, b_n \in \mathbb{R}$ such that $a_n \to a \in \mathbb{R}$. Prove that

$$\liminf(a_n + b_n) = a + \liminf b_n.$$

b. Let f, f_n be integrable functions. Assume $f_n(x) \to f(x)$ a.e. and $\int |f_n|\,dx \to \int |f|\,dx$. Prove that $\int |f_n - f|\,dx \to 0$. (Hint: Apply Fatou's lemma to $|f| + |f_n| - |f - f_n|$.)