Homework 6

1. Find all solutions of

   a. $e^z = -i$.

   b. $\sin z = 0$.

   c. $\operatorname{Log} z = 1 + i$.

2. Suppose that f is holomorphic in the open unit disk $D(0;1)$.

   a. Prove that if $\operatorname{Re} f$ is constant on $D(0;1)$, then f is constant.

   b. Prove that if e^f is constant on $D(0;1)$, then f is constant.

3. Let $a \in \mathbb{C}$ with $|a| < 1$, Prove

$$\left| \frac{a - z}{1 - \overline{a}z} \right| = 1 \Leftrightarrow |z| = 1.$$

4. Suppose $P(z) = a_0 + a_1 z + \cdots + a_n z^n$, where $n \geq 1$ and $a_0 \geq a_1 \geq \cdots a_n > 0$. Prove that the zeros of $P(z)$ all lie outside the open unit disk $D(0;1)$. (Hint: Look at $(1-z)P(z)$ and show that $(1-z)P(z) = 0$ implies that $a_0 = (a_0 - a_1)z + \cdots + (a_{n-1} - a_n)z^n + a_n z^{n+1}$ and show that this is impossible for $|z| < 1$.)