

Homework 5

1. Express in the form $a + bi$.

a. $(1 + i)^{20}$.

b. $\frac{1-2i}{2+i}$.

2. Solve $z^2 - 4z + (4 + 2i) = 0$.

3. **a.** Prove $|z - w|^2 + |z + w|^2 = 2(|z|^2 + |w|^2)$ for all $z, w \in \mathbb{C}$ (Parallelogram identity).

b. Interpret the identity geometrically.

4. Prove that

$$1 + \cos \theta + \cdots + \cos n\theta = \cos \left(\frac{n\theta}{2} \right) \cdot \frac{\sin \frac{(n+1)\theta}{2}}{\sin \frac{\theta}{2}},$$

provided $\sin \frac{\theta}{2} \neq 0$.