Homework 4, Additional Problem.

(1) Let $(X, d)$ be a compact metric space and $f : X \to X$ a mapping such that $d(f(x), f(y)) < d(x, y)$ for all $x \neq y$.

    **a.** Show that there exists $x_0 \in X$ such that $f(x_0) = x_0$. (Hint: Consider $\inf\{d(x, f(x)) : x \in X\}$, show that it is a minimum and that this minmum must be equal to zero.)

    **b.** Show that the fixed point of part a) is unique.