Extra problems Homework 4.

(1) Let $f, g : \mathbb{R} \to \mathbb{R}$ be uniformly continuous functions. Assume that both f and g are bounded. Prove that the product fg is uniformly continuous.

(2) A function $f : \mathbb{R} \to \mathbb{R}$ is periodic, if there exists a $c \in \mathbb{R}$ such that $f(x+c) = f(x)$ for all $x \in \mathbb{R}$. Prove that a continuous periodic function on $\mathbb{R}$ is bounded and uniformly continuous.