Homework 4.

(1) Problem 30-1

(2) Problem 30-8

(3) Problem 30-9

(4) Let $f_n : \mathbb{R} \to \mathbb{R}$ be defined by $f_n(x) = x^{2n} \left(1 + x^{2n}\right)^{-1}$. Prove that the pointwise limit $\lim_{n \to \infty} f_n(x)$ exists for all $x \in \mathbb{R}$, but that f is discontinuous at $x = \pm 1$.

(5) Define $f_n(x) = nxe^{-nx^2}$ for $x \in \mathbb{R}$. Prove that $f_n(x) \to 0$ pointwise on $\mathbb{R}$, but not uniform on any interval containing 0. (Hint for the second part: consider the points $x_n = \frac{1}{\sqrt{n}}$.)